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# A NEW DETERMINATION OF THE RATIO OF THE ELECTROMAGNETIC TO THE ELECTROSTATIC UNIT OF ELECTRICITY.

[Continued.]

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## VII. DETERMINATION OF CAPACITIES IN ELECTROMAGNETIC MEASURE.<sup>13</sup>

In the electromagnetic measurement of the capacity two distinct methods were employed, viz: (1) The method described by Maxwell;<sup>14</sup> (2) the differential galvanometer method. For the latter method connections were so made that we could readily measure either the charging or the discharging current.

### 39. MAXWELL'S BRIDGE METHOD.

As is well known this method consists in replacing one of the resistances in a Wheatstone bridge by a condenser and a suitable device for rapidly and regularly charging and discharging the condenser. Maxwell originally suggested a periodic reversal of the terminals of the condenser, but this is not so good, mechanically, as the periodic charging and discharging by touching one terminal of the condenser *P* (Figs. 29, 30) alternately to *R* and *S*. Then, by a suitable adjustment of the frequency of the charging, and of the magnitude of the resistances of the net, a balance of the galvanometer can be obtained. When this is done the relation between the electromagnetic capacity and the resistance will be<sup>15</sup>

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<sup>13</sup> We are under obligations to Dr. F. W. Grover and Mr. H. D. Babcock for assistance in some of the experimental work, and to Mr. G. W. Vinal and Miss G. C. U. McDermut for assistance in preparing the results for publication.

<sup>14</sup> Electricity and Magnetism, Vol. II, § 776.

<sup>15</sup> J. J. Thomson, Phil. Trans., **174**; 1884. Proc. Roy. Soc. **35**, p. 346; 1883. Rosa and Grover, this Bulletin, **1**, p. 153; 1905.

$$C = \frac{a \left[ 1 - \frac{a^2}{(a+c+g)(a+b+d)} \right]}{ncd \left[ 1 + \frac{ab}{c(a+b+d)} \right] \left[ 1 + \frac{ag}{d(a+c+g)} \right]} = \frac{aF}{ncd}$$

where

$$F \equiv \frac{1 - \frac{a^2}{(a+c+g)(a+b+d)}}{\left[ 1 + \frac{ab}{c(a+b+d)} \right] \left[ 1 + \frac{ag}{d(a+c+g)} \right]}$$

The letters stand for the values of the resistances (in international ohms) of the branches against which they are placed in Figs. 29, 30;  $n$  is the number of charges of the condenser per second.  $F$  is a correction factor differing but little from unity, in our work only a few parts in 100,000.

In the case of guard ring condensers it is necessary that the guard ring be charged to the same potential as the main condenser. This

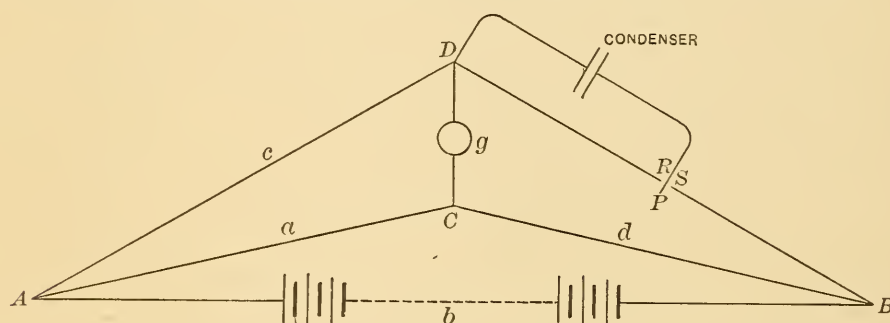


Fig. 29.—Maxwell Bridge for Measuring the Capacity of a Condenser.

The points P, R, S are joined to  $B_3$ ,  $B_2$ ,  $B_1$ , respectively, of the commutator, Figs. 32 and 32a.

condition is readily obtained by constructing, as shown in Fig. 30, a second Wheatstone net exactly duplicating the main net and containing the same battery; for, if the two nets contain the same resistances branch for branch, the maximum potential of  $D'$  must be the same as the maximum potential of  $D$ , and this maximum potential is the potential to which the condenser and ring are charged, provided neither of the charging points P and P' breaks contact with the corresponding charging (or discharging) points R, R' (or S, S') before *both* condenser and ring are fully charged (or discharged). This condition is easily fulfilled. It is also necessary,



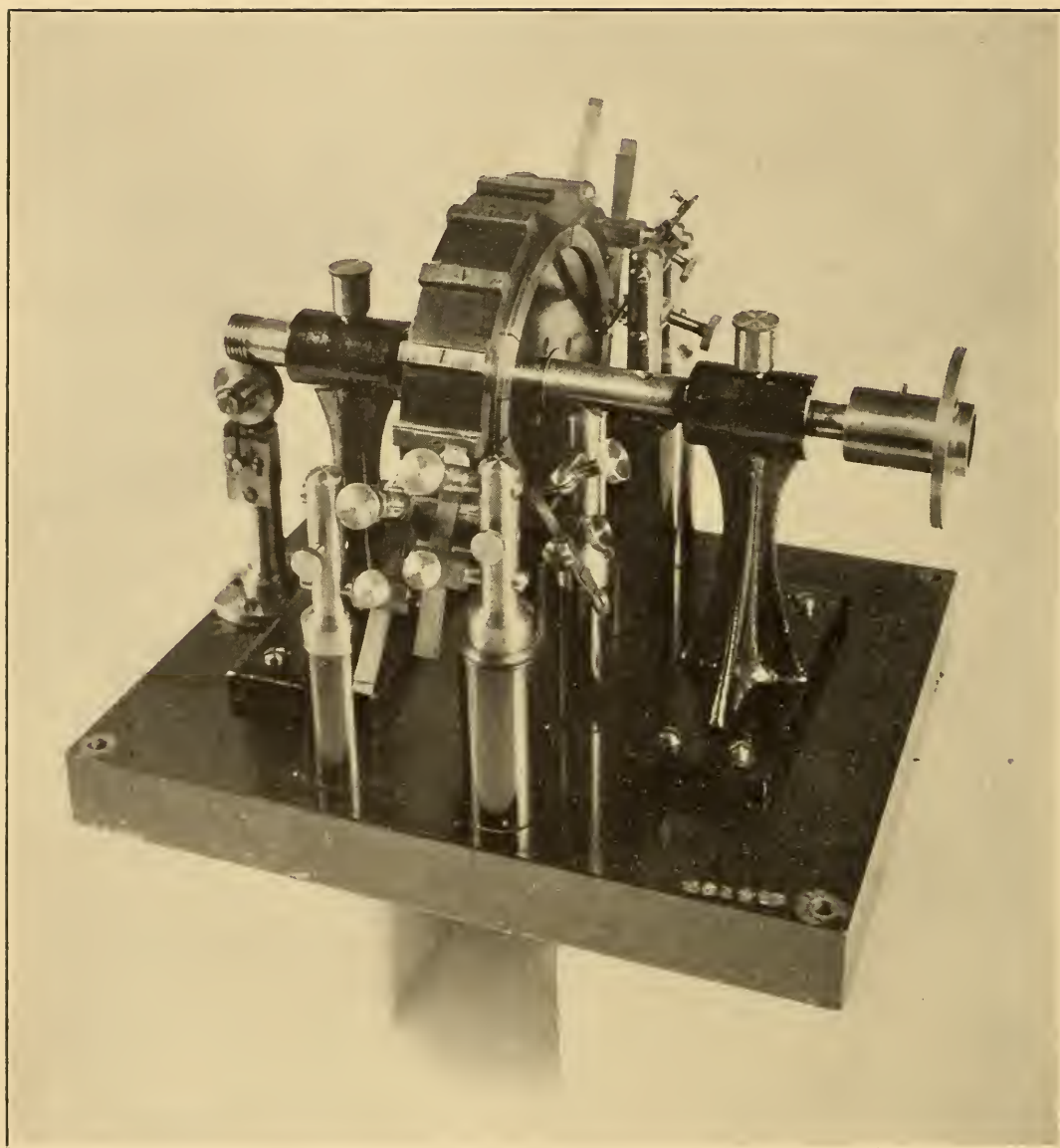


Fig. 31.—*Rotating Commutator.*

for exact work, for the continuous plate of the condenser to be connected to the point B instead of D.

The charging and discharging of the condenser is effected by means of a rotating commutator similar to the commutator described in this Bulletin, vol. 1, p. 172, except that there are sixteen contact pieces, and the ring of brass to which the contact pieces are connected is divided into sixteen segments  $T_1, T_2$ , etc., insulated from one another so that but one segment and contact piece is charged at a time. This materially lessens the capacity of the commutator, and allows the guard ring of the condensers to be charged by the

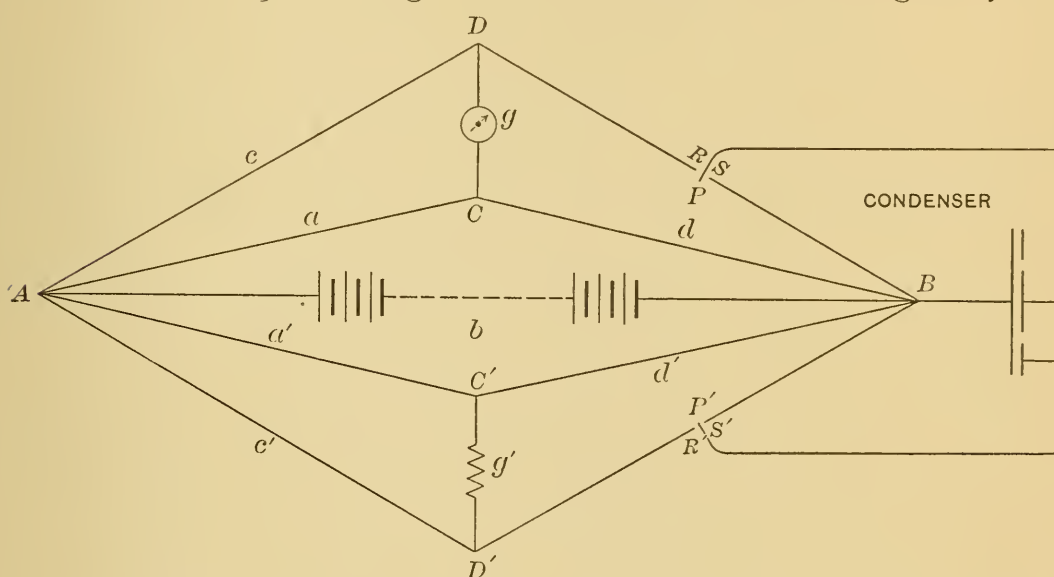


Fig. 30.—Double Maxwell Bridge, for Measuring the Capacity of a Guard Ring Condenser.

The corresponding arms  $a, a'; c, c'$  etc., have identical resistances, and the potentials of D, D', and C, C' are therefore the same.  $P', R', S'$  are joined to the second set of brushes of the commutator  $B_6, B_5, B_4$  of Fig. 32a.

same commutator as that employed for the main condenser, the charging of the ring being effected by a second set of three brushes,  $B_4, B_5, B_6$ , placed diametrically opposite those which serve for the main condenser. The insulation across the commutator was frequently tested and was always found to be as nearly perfect as could be desired; its resistance was of the order of a million megohms. The contact pieces  $S_1, S_2$ , etc., of the commutator are of phosphor-bronze, and the brushes  $B_1, B_2$ , etc., are of thin brush copper clamped between springs reaching to within about 4 mm of their tips; steadying screws with dampers reduced the vibration. As shown in Figs. 31, 32, and 32a, the phosphor-bronze segments project

beyond the surface of the ebonite disk of the commutator so that the tips of the brushes  $B_1$ ,  $B_2$ , etc., are *air insulated*, except when in contact with the metal segments. The commutator was ordinarily driven at the rate of 1,200 to 1,700 revolutions per minute. A counter  $C$  made contact and recorded every fiftieth revolution on the chronograph.

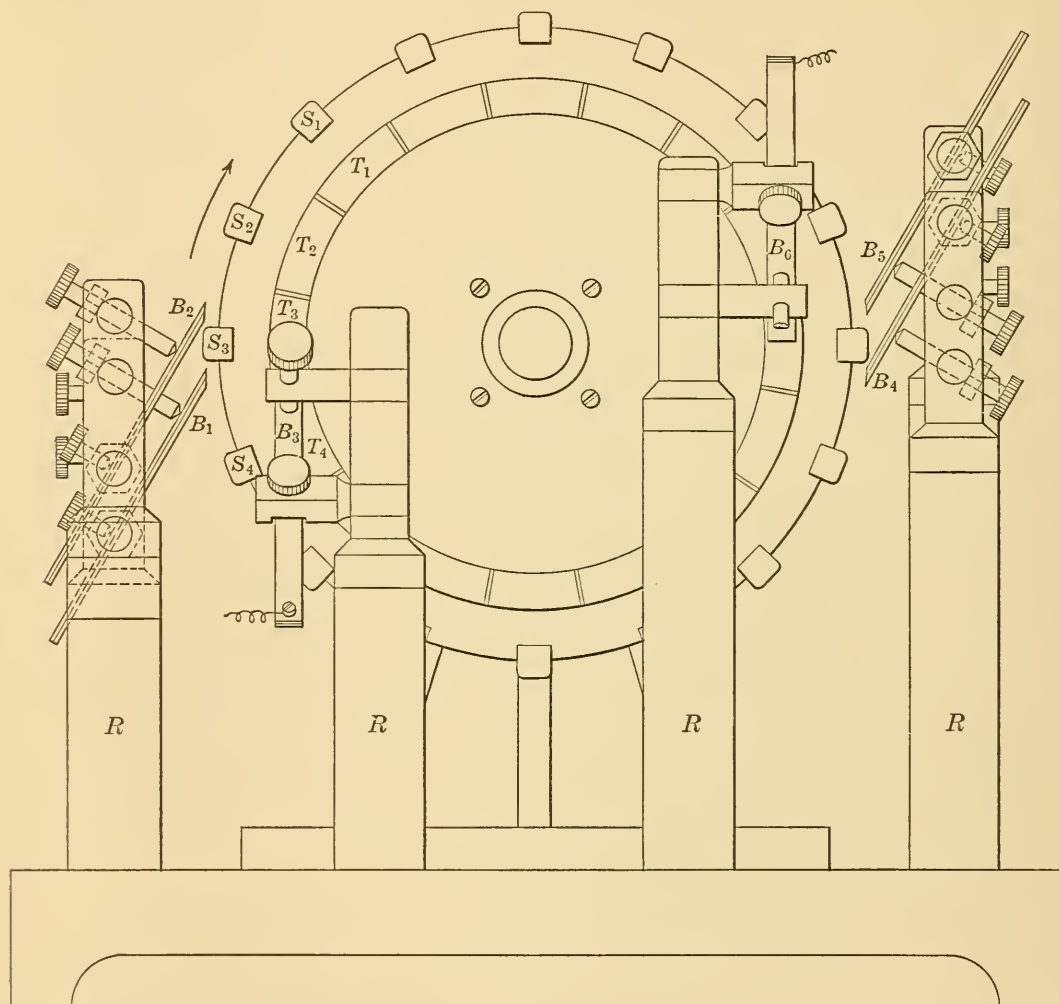


Fig. 32.—Rotating Commutator for Charging and Discharging the Condensers.

The six metal brushes are mounted on six ebonite columns, three on each side. The condenser is joined to  $B_3$ , and then to  $T_3$  and the segment  $S_3$  when the commutator is in the position shown in the figure. The condenser is charged when  $S_3$  touches the brush  $B_1$  and discharged when it touches  $B_2$ . It is charged again an instant later when  $S_4$  touches  $B_1$ , etc. The guard ring of the condenser is joined to  $B_6$ , and is similarly charged and discharged by the brushes  $B_4$  and  $B_5$  simultaneously with the charging and discharging of the main condenser. The brushes are carefully adjusted so that the interval between charging and discharging shall coincide as nearly as possible for the condenser and its guard ring.

## 40. DIFFERENTIAL METHOD.

In this method the charging or the discharging current is passed through one coil of a differential galvanometer, while through the other coil is passed a steady current shunted off from a portion of a high resistance which permanently connects the terminals of the

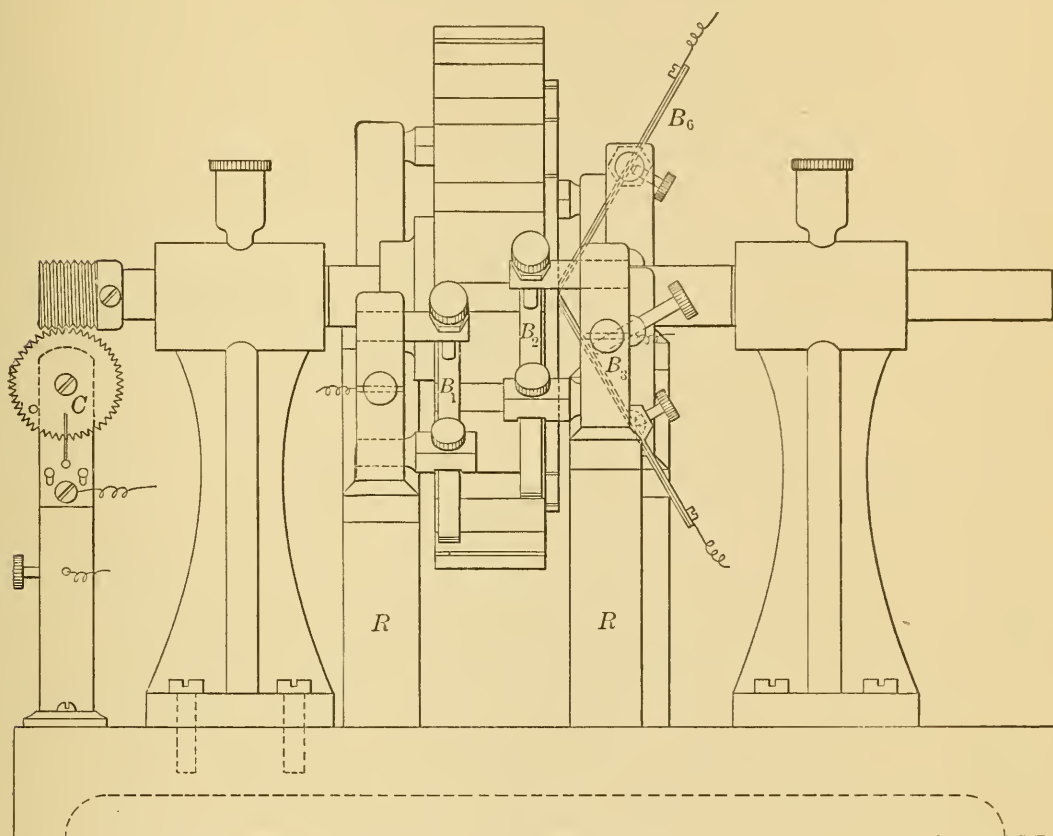


Fig. 32a.—*Rotating Commutator; Side View.*

battery employed to charge the condenser. The continuous lines in Fig. 33 show the connections in case we wish to send the discharge current through the galvanometer. If we wish to send the charging current through it, we disconnect BE and connect B'E'. If the condenser has a guard ring the connections for measuring the discharge current are shown by the continuous lines in Fig. 34. To measure the charging current we disconnect JK and connect J'K; and interchange the connections at the posts S and R also at the

posts  $S'$  and  $R'$ . If as before we denote the resistances of the various branches by the small letters written near them, and call  $E$  the emf.

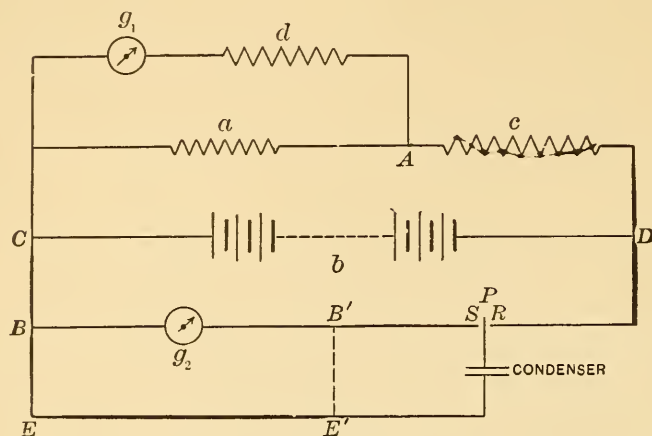


Fig. 33.—Arrangement of Resistances, Galvanometer Coils, etc., for Measuring the Capacity of a Condenser by the Differential Method.

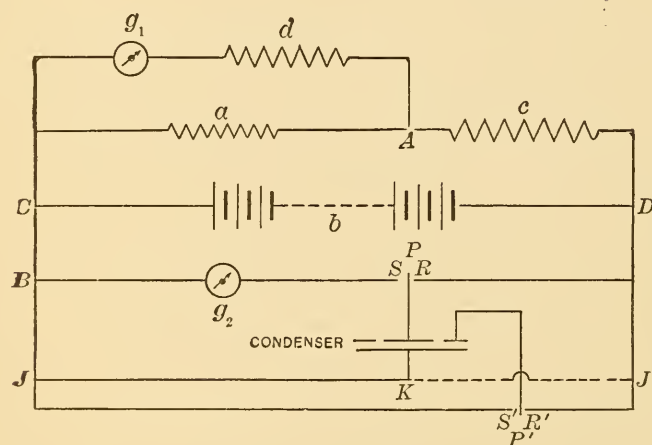


Fig. 34.—Arrangement of Resistances, Galvanometer Coils, etc., for Measuring the Capacity of a Guard Ring Condenser by the Differential Method.

Same as Fig. 33, except for connections of guard ring to second set of commutator brushes, etc.

of the battery, we find the following equations for the differential method:

$$\text{Current through AD} = i = \frac{E}{c + \frac{a(d+g_1)}{a+d+g_1}} = \frac{E(a+d+g_1)}{c(a+d+g_1) + a(d+g_1)}$$

$$\begin{aligned}\text{Current through } g_1 = i_1 &= \frac{ai}{a+d+g_1} = \frac{Ea}{cd \left[ 1 + \frac{a+g_1}{d} + \frac{a}{cd} (d+g_1) \right]} \\ &= \frac{Ea}{cd \left[ 1 + \frac{g_1}{d} + a \left( \frac{1}{d} + \frac{1}{c} + \frac{g_1}{cd} \right) \right]} \\ &= \frac{Ea}{cd [1 + A_2]}\end{aligned}$$

where

$$A_2 \equiv \frac{g_1}{d} + a \left[ \frac{1}{d} + \frac{1}{c} + \frac{g_1}{cd} \right]$$

The current through  $g_2 = i_2 = nCE$ ,  $n$  being the number of charges per second. Hence for balance when  $i_1 = i_2$

$$nC = \frac{a}{cd(1 + A_2)}$$

or

$$C = \frac{a}{ncd(1 + A_2)}$$

This formula is strictly analogous to the one obtained for the bridge method. However, the quantity  $A_2$  is here not small, but may amount to several per cent of the whole. The galvanometer coils were shunted so as to make the system aperiodic and at the same time to make the coils magnetically equivalent, so that, when a considerably larger current than any used in measuring capacities is sent through the two coils in opposite directions, the deflection produced is too small to be observed. This equality test was made frequently. The resistances  $g_1$  and  $g_2$  are the effective resistances of the coils of the galvanometer with their respective shunts. The resistances  $g_1$  and  $g_2$  were measured whenever the galvanometer was used.

A great advantage of the differential method is that it gives a direct check against the insulation of the armatures of the condenser. If there is any appreciable leakage between the plates of the condenser, either through their supports or by way of the commutator and conductors leading to it, then the capacity of the condenser as measured by the use of the charging current must of necessity be greater

than the capacity as measured by means of the discharging current. No certain difference of this kind was ever noticed. This method, however, does not give a check against leakage from the brush S to

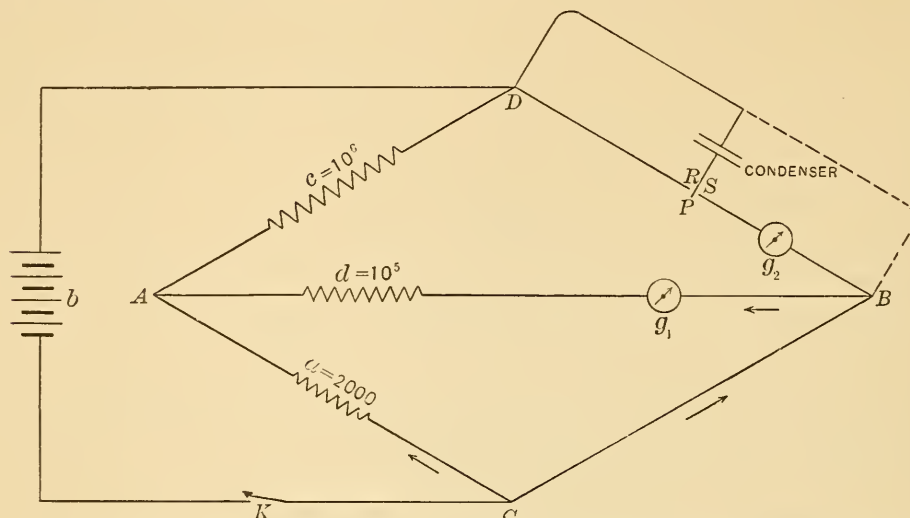


Fig. 35.—Arrangement of Resistances as in Fig. 33 for the Differential Method, Showing the Resemblance to the Maxwell Bridge.

The arm BC has zero resistance.

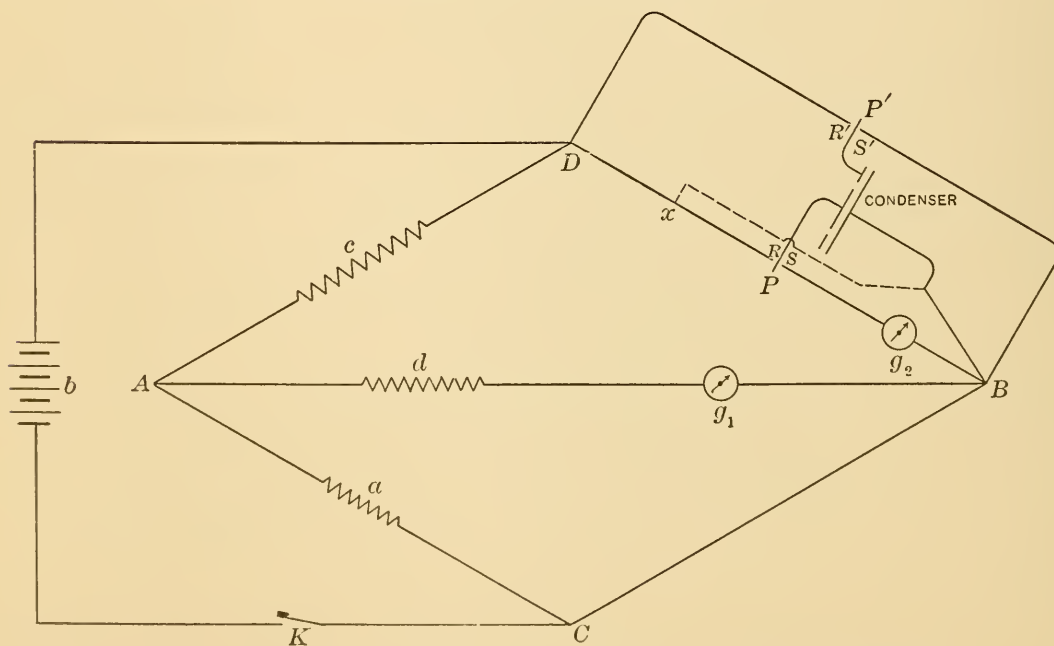


Fig. 36.—For Measuring Guard Ring Condenser by Differential Method.

Same arrangement of resistances as shown in Fig. 34.

the brush R. The resistance between these points was, however, frequently measured and was always of the order of a million megohms.

Another advantage of this method is that the resistance in the charge or discharge circuit is always rather small, being merely that of the galvanometer coil  $g_2$ , which is about 80 ohms. It is the same whatever the capacity, so that the duration of contact of the brushes in this method might be shorter, if desired, than in the bridge method. However, they were always adjusted by the same criterion as described on page 552.

The connections for the differential method are also represented in Figs. 35 and 36, which are exactly equivalent to Figs. 33 and 34.

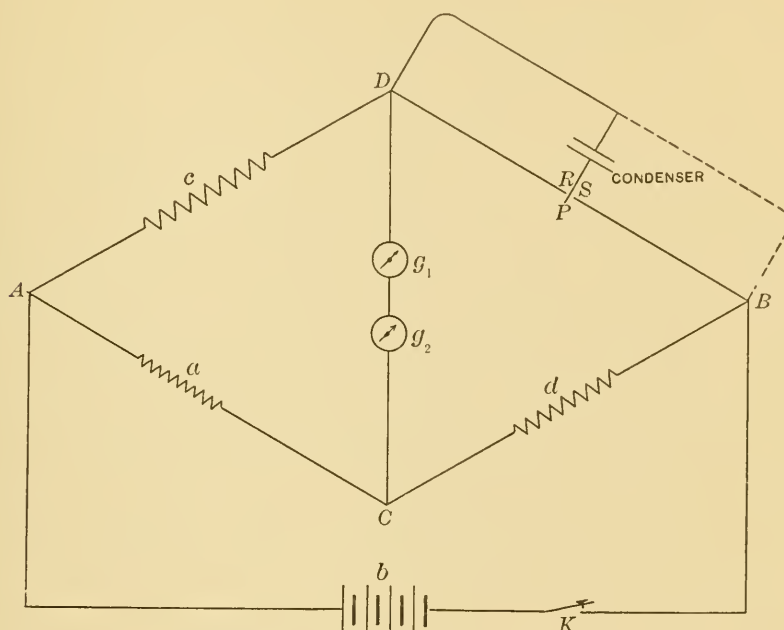


Fig. 37.—Showing how the Maxwell Bridge Arrangement is Obtained by Rearranging the Resistances and Galvanometer Coils of the Differential Method, Fig. 35.

To change the connections shown in Fig. 35 into the proper connection for the bridge method we have only to (1) interchange the battery and galvanometer (putting both halves of the galvanometer in series) and (2) put the resistance  $d$  in the arm CB (see Fig. 37). We now have a Maxwell bridge consisting of the same resistances, and it will be balanced when a small change is made in  $a$ , corresponding to the difference in the correction factors,  $F$  in the one case and  $1 + A_2$  in the other. This change is so quickly made that it is very convenient to compare measurements by the two methods, especially in the case of the spherical condensers where, owing to the absence of a guard ring, a single bridge is used.

## 41. COMPLETENESS OF CHARGE.

If the duration of contact of the commutator brushes is not sufficient, the condenser will not be completely charged or completely discharged, hence, the measured capacity will be less than the true capacity in the ratio of  $(1 - e^{-\frac{t_1}{CR_2}})$  to 1. Where  $t_1$  is the duration of contact of the brushes,  $C$  is the capacity, and  $R_2$  is the resistance of the network of conductors joining the points B and D of the bridge.<sup>16</sup> If, however,  $d$  is very large (in our case it is  $10^5$  ohms),

then  $R_2$  may be replaced by its approximate value<sup>17</sup>  $R'_2 = b + \frac{c(a+g)}{c+a+g}$

For the large sphere and the high resistance galvanometer  $C = 5.5 \times 10^{-11}$  farads,  $a < 3,500$  ohms,  $b = 100$  ohms  $c = 10^6$  ohms,  $g = 1,000$  ohms,  $\therefore R'_2 C < 2.5 \times 10^{-7}$  seconds.

The highest speed ever reached during the work was 1,960 revolutions per minute, the circumference of the commutator over contact pieces is 40 cm, the width of each contact piece is 0.5 cm; hence, the time required for a contact piece to pass any given point is  $t = \frac{0.5}{40} \times \frac{60}{1,960} = 3.8 \times 10^{-4}$  seconds. If the brush makes contact over

$\frac{1}{n}$  of the total width of a contact piece  $t_1 = \frac{t}{n} = \frac{3.8}{n} \times 10^{-4}$

$$\text{Hence, } e^{-\frac{t_1}{CR_2}} = e^{-\frac{3.8 \times 10^{-4}}{2.5n \times 10^{-7}}} = e^{-\frac{1.5 \times 10^3}{n}}$$

If we allow an error of so much as 1 in 10,000 we can determine the maximum allowable value of  $n$  from the equation

$$e^{-\frac{1.5 \times 10^3}{n}} = 10^{-4} \\ \therefore n = 160$$

Hence, for this case the error due to this cause will amount to only 1 in 10,000 even when the brushes never make contact over more than  $\frac{1}{160}$  of the total width of a commutator segment. By means of the tests with the millimeter, as described on page 552, it was estimated that at no time, when the action of the commutator was

<sup>16</sup> See Maxwell, Elec. and Mag., Vol. II, § 777.

<sup>17</sup> This Bulletin 1, p. 169; 1905.

regarded as good enough to give reliable results, did the brushes make contact over less than 0.75 of the width of the segment, and usually (owing to the finite length of the portion of the brush making contact) the contact extended over the entire width of the segment.

For the largest capacity practically obtainable with the plate condensers used, we have  $C = 119 \times 10^{-11}$  farads,  $a = 14,000$ ,  $g = 1,000$ ,  $c = 0.250 \times 10^6$ ,  $b = 100$ ;  $\therefore CR_2 = 1.7 \times 10^{-5}$ . Hence for an accuracy of 1 in 10,000 we must have  $n$  no greater than is given by the equation

$$e^{-\frac{3.8 \times 10^{-4}}{1.7n \times 10^{-5}}} = 10^{-4} \\ \therefore n = 2.44$$

Hence, even in this extreme case, which was never reached except in the preliminary work, the margin of safety would appear to have been sufficient.

All of this applies directly to the case in which the brushes are working smoothly and possess but little free vibration. This condition can always be obtained by properly adjusting the springs between which the copper brushes are held and the angles at which the segments meet the brushes. Hence for condensers without guard rings this correction can be made vanishingly small. Unfortunately, however, in the case of guard-ring condensers it is often impracticable to get *both* sets of brushes running smoothly simultaneously and still have them in proper phase relation. Under these conditions the brushes are set in rather violent free vibration, which renders the duration of contact uncertain and gradually bends the brushes so that the contact becomes worse and worse. This condition can be improved, but not completely eliminated, by placing soft rubber dampers on the backs of the brushes and extending nearly to their tips. These rubbers are gilded so as to eliminate any irregular creeping of electricity over them. Having got the brushes adjusted as well as possible, they are screwed down against the commutator until they seem to make as long a contact as is possible without harmful vibration. The bridge is then balanced, after which an extra resistance of 10,000 or 20,000 ohms is introduced seriatim in the branches DR, SB, BS', R'D'. If the balance

is not affected by this process we are justified in concluding that the brushes make sufficiently good contact. If the balance is affected by this process the brush or brushes at fault are readjusted. Having thus got the brushes into what is known to be a good adjustment, it is possible to obtain an empirical and more expeditious method of testing the functioning of the brushes. This consists in placing a milliammeter and storage cell in series with P and R, D and B being disconnected from R and S, respectively. The reading of the milliammeter gives a measure of the average duration of contact, and the constancy of its reading is an index to the regularity of the contact. Similarly the other three contacts are tested. After a little experience in this way under conditions which are known to be good, or bad, as the case may be, one learns what reading of the ammeter to expect under different conditions, after which it serves as a very ready and satisfactory method for determining the behavior of the brushes.

#### 42. RESISTANCES.

The resistances employed were as follows: In the arm *a*, at various times were used, (1) a box by Otto Wolff, No. 2470; resistances range from 0.1 ohm to 50,000 ohms, the coils are of manganin, are wound noninductively upon metal spools, and were oil immersed. (2) An exactly similar box, No. 2471. The coils of these two boxes are shortcircuited, when desired, by conical plugs in the usual way. (3) A dial box, also by Wolff, No. 3087, having a range from 0.1 ohm to 10,000 ohms. These coils are similar to the coils of the other boxes, but were not oil immersed.

Arm *c* always consisted of one or other of two similar boxes by Siemens and Halske, No. 15657 and No. 16228. Each consists of a series of ten manganin coils, nominally of 100,000 ohms each. The coils are embedded in paraffin into which holes are drilled for the insertion of thermometers. The total resistance of No. 15657 is approximately 1,000,880 international ohms at 20° C, and that of No. 16228 is 1,001,180 international ohms at 20° C.

For the larger capacities the megohm in *c* has its two halves connected in parallel, making the resistance nominally 250,000 ohms.

Arm *d* consisted of one of the following: (1) A box by Otto Wolff, No. 2620, consisting of a series of ten manganin coils of approxi-

mately 10,000 ohms each. The coils are always oil immersed. (2)  
An exactly similar box, No. 2621, also oil immersed.

Thus, the resistance of  $d$  is always nominally 100,000 ohms, and the resistance of  $c$  is always nominally 1,000,000 or 250,000 ohms.

The resistance of  $a$  is varied so as to balance the bridge.

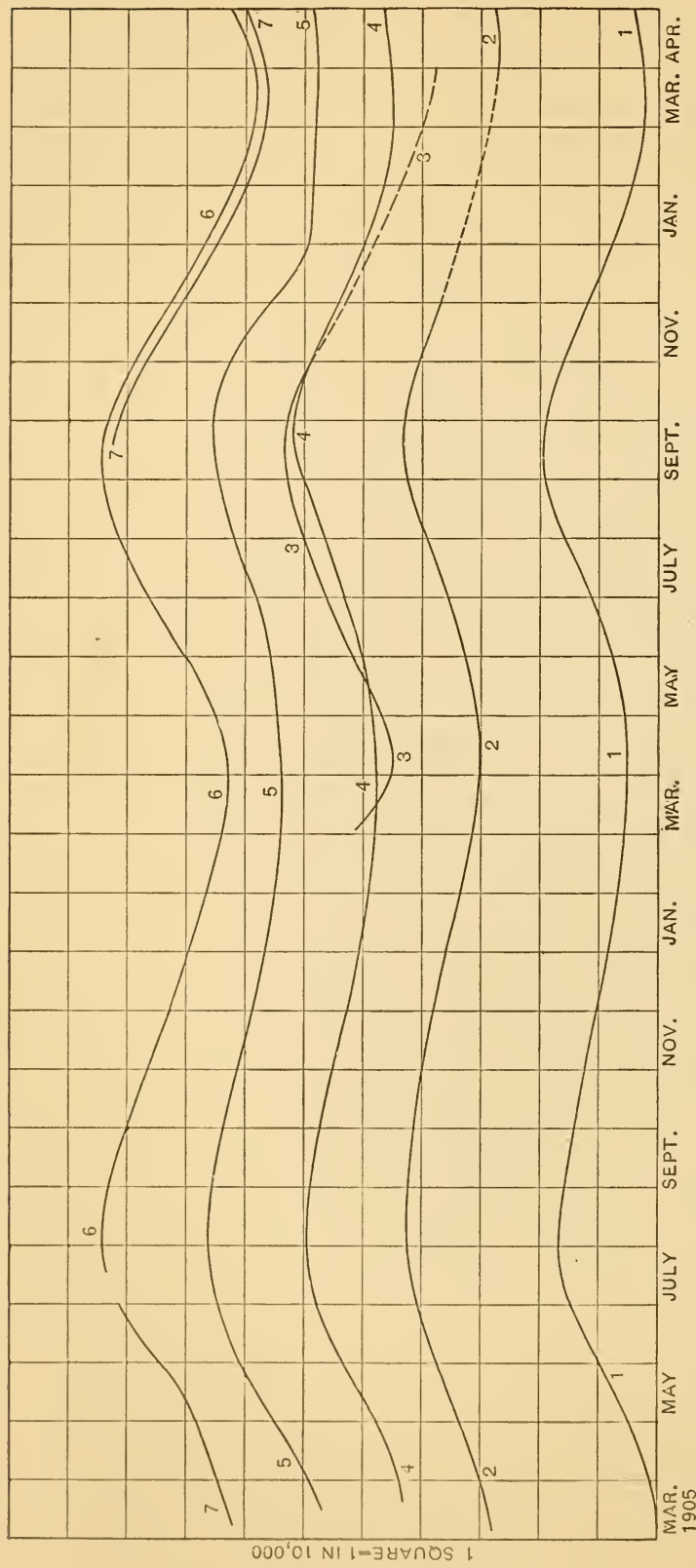
These coils above 500 ohms were intercompared almost daily when in use, and comparisons were made at frequent intervals with the standards of this Bureau. The intercomparison was performed as follows: The 10,000 ohm combination, obtained by connecting the coils of the megohm in parallel, was compared with the 10,000 ohm combination obtained by connecting the first nine coils of the 100,000 ohm box in square array, three in series and three such groups in parallel; then the coils in the 100,000 ohm box were connected in parallel and the 1,000 ohms thus obtained was compared with the 1,000 ohm coil of the box in arm  $a$ . The other coils of  $a$  were compared with the 1,000 ohm coil in the usual manner. Thus, we get all of the resistances in terms of the 1,000 ohm coil in the box in arm  $a$ . This and other coils are frequently compared with the standards. This method of comparison is justified by the fact that all the coils in any one of the decade boxes considered have nearly the same value; the error thus introduced in the case of these boxes does not amount to more than 2 parts in 1,000,000.

#### 43. STANDARDS OF RESISTANCE.

The standard resistance employed in this work is the mean value of the resistances of 11 one-ohm and 7 tenth-ohm coils belonging to this Bureau. Groups of these coils have recently and at various times in the past been compared with the standards of the Physikalisch-Technische Reichsanstalt, so that we feel confident that the value of this mean is known in terms of the international ohm with an accuracy in excess of that required for the present work.

#### 44. SEASONAL CHANGES OF THE RESISTANCES.

By the process of intercomparison of the various coils and the frequent comparison of certain of them with the standards as just described, we obtain data from which, after proper temperature corrections have been applied, we are able to construct curves showing the relation between the corrections to be applied to the different



coils at some constant temperature (say 20° C) and the time. Doing this we obtain the very surprising and interesting group of curves shown in Fig. 38. The ordinates represent the correction in parts per 10,000 to be added to the nominal value of the coil in order to obtain its true resistance in terms of our standard ohm, the origin being arbitrary. The abscissæ represent the time, each division corresponding to one month. It will be noticed that every curve has a minimum between February and April, then rises gradually to a maximum in July and August, then decreases to a minimum again the following February and April. This unanimity is so much the more surprising when we consider the great difference in the coils under study. The behavior of the

1,000-ohm coil of No. 2470 is shown in curve No. 1;

1,000-ohm coil of No. 2471 is shown in curve No. 2;

1,000-ohm coil of No. 3087 is shown in curve No. 3;

Combination of (500+200+200+100) ohm coils of No. 2470 is shown in curve No. 4;

Combination of (500+200+200+100) ohm coils of No. 2471 is shown in curve No. 5;

Ten 10,000-ohm coils of No. 2620, connected in parallel, is shown in curve No. 6;

Ten 10,000-ohm coils of No. 2621, connected in parallel, is shown in curve No. 7.

Boxes No. 2470 and No. 2471 are probably of the same age and have been subjected to the same treatment, except that No. 2471 has been at constant humidity since December, 1906. The coils of these boxes have been in oil since February, 1905. Box No. 3087 is a new box of a different type from No. 2470. It was purchased in the winter of 1905-6; its coils are not oil immersed. Boxes No. 2620 and No. 2621 are probably of the same age and have undergone similar treatment; the coils are oil immersed. Thus four of the seven curves are entirely unrelated to one another. These are

I. Curves 1 and 2, behavior of certain 1,000-ohm coils in oil;

II. Curve 3, behavior of a different 1,000-ohm coil in air;

III. Curves 4 and 5, behavior of coils under 1,000 ohms in oil;

IV. Curves 6 and 7, behavior of coils of 10,000 ohms in oil.

Throughout this time the boxes have all been kept at laboratory temperatures which have fluctuated from about 15° to 30° C, though

the temperature was rarely under  $19^{\circ}$  or over  $27^{\circ}$  C; they have not been heavily loaded, and in no sense can their treatment, other than by the weather, be said to have been of a periodic character. The variations can not be due to leakage produced by moisture condensed upon portions of the box, for the resistances of the coils are highest during July and August, when the humidity in this region is the greatest; and it is much too great to be explained by an uncertainty in the temperature coefficients.

It is probably due to strains produced by the expansion of the shellac, in which the coils are embedded, as moisture is absorbed, the oil not protecting the coils completely, even when they are completely submerged. This question has been under investigation in this laboratory for some time and will form the subject of another paper. These curves emphasize the necessity of very frequent comparisons of resistances with the standards when an accuracy of 1 in 10,000, or better, is desired.

The temperature coefficients of the other coils employed were not determined, but they were almost daily compared at laboratory temperatures with the coils of which we know the coefficients. This enables us to correct our results for any variations in the resistances used.

#### 45. GALVANOMETERS.

Two galvanometers, both of the D'Arsonval type, were used. One was a 1,000-ohm galvanometer by Sullivan. It had a sensibility of 1,000 mm, at 1 meter scale distance, per microampere. Its period is about nine seconds, damping on open circuit is very slight, and its zero is very constant. It was used at a scale distance of 2.5 meters, and was shunted by a noninductive resistance of 10,000 ohms. This was just sufficient to make the instrument aperiodic. Its inductance, including the leads, was found to be 66 millihenrys; hence its time constant is negligible as compared with the duration of contact of the commutator brushes.

The other galvanometer was a fine differential D'Arsonval made by the Weston Electrical Instrument Company. When the coils are connected in series, it has a sensibility equal to that of the Sullivan. Each coil has a resistance of about 82 ohms, its period is about five seconds, its damping is slight, and its zero is extraordinarily con-

stant. When each coil is shunted with 2,000 ohms the instrument is nearly aperiodic. The two coils were very nearly equal in their effects, and the two circuits were made equal by a proper adjustment of the shunts. The inductance of the coils is small, but was not measured.

The only remaining question to be considered is that of the division of the current between the galvanometer and its shunt. In deducing our formulæ we assume that the intermittent current from the condenser divides between the galvanometer and its shunt in the same proportion as a constant current does; that is, in inverse proportion to their resistances. We have seen that the time constants of the galvanometers are small as compared with the duration of contact of the commutator brushes, hence the intermittent current starts from zero, rises to a maximum, and falls to zero again. Hence the momentum of the field is the same at the end of the process as it is at the start; hence the integral over this period of the applied emf. must be equal to the integral over the same time of the total dissipative force in this branch. Now the only appreciable dissipative force is  $ir$ , where  $i$  is the current and  $r$  is the resistance. Hence

$$\begin{aligned}\int_0^T V dt &= \int_0^T i_1 r_1 dt = \int_0^T i_2 r_2 dt \\ &= Q_1 r_1 = Q_2 r_2\end{aligned}$$

Hence, the quantity of electricity carried is divided between the two circuits in the same proportion as in the case of constant currents.

Here it may be mentioned that at one time both galvanometers, each separately shunted, were connected in series and used. The high-resistance galvanometer had an inductance probably twenty times that of the low-resistance galvanometer. It was found that they were simultaneously balanced, which could not be the case if the division of the current between the galvanometer and the shunt depended on the inductance of the former.

#### 46. REGULATION AND MEASUREMENT OF SPEED.

The commutator was driven by a small motor-generator set (0.5-kw motor and a 0.3-kw alternator), which was driven by the cur-

rent from a large storage battery. On each end of the motor-generator shaft was mounted a heavy fly wheel, and the alternator carried a load composed of two 16-candlepower lamps to steady the speed. In the armature circuit of the motor was a carbon rheostat by which the speed could be very delicately controlled. At other times the speed was regulated by gentle friction applied to the rim of one of the fly wheels on the shaft of the motor. The method of procedure has already been described in this Bulletin<sup>18</sup>. The resistance in the bridge having been suitably adjusted, the speed of the motor was varied until a balance was obtained, as indicated by the galvanometer; then a chronographic record of the speed was obtained while the rheostat was manipulated so as to keep the galvanometer continuously as near zero as possible. Under favorable conditions it is possible to hold the speed so nearly constant that the values found for it from successive three-minute intervals of the record will differ from one another by only a few parts in 100,000.

The time was given by one or other of two chronometers which were occasionally rated by comparison with the standard Riefler clock of this Bureau. The latter was daily compared with mean solar time, as determined at the Naval Observatory. The rates of the chronometers were found to be subject to a very slight linear change, an allowance for which is made in the reductions.

In the last part of the work another method of procedure was adopted, which is more expeditious than that just outlined. As stated above, the commutator was attached to one extremity of the axis of a small motor-generator set. To the other extremity of this axis was attached a second commutator making four contacts on each brush per revolution. This commutator, which necessarily has always the same speed as the other, together with a mica condenser (0.2 mf) formed one arm of an auxiliary Wheatstone bridge. By a proper adjustment of resistances the two bridges were balanced simultaneously for a certain speed of the machine. If the capacity of the air condenser is slightly changed (the resistance in the main bridge remaining unchanged), it will be necessary to change the speed, and consequently the resistance in one of the arms of the auxiliary bridge, in order to restore the simultaneous balance of the two bridges. Except for sign, the percentage change in speed,

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<sup>18</sup> Vol. 1, p. 176.

which is equal to the percentage change in the capacity of the air condenser, is equal to the percentage change in the resistance in the auxiliary bridge, provided the changes are not too great. Hence, we may proceed as follows: With the machine running at a convenient speed, the auxiliary bridge is balanced as nearly as possible. Then the observer at the auxiliary bridge changes the speed of the motor slightly, so as to give an exact balance, and maintains the balance until a sufficiently long chronograph run has been made. This determines the speed which corresponds to the existing values of the resistances in the auxiliary bridge. While this is being done the observer at the main bridge determines the values of the resistances  $\alpha_1$  and  $\alpha_2$  in the arm  $\alpha$  of his bridge, which will give the nearest possible balance (1) for the condenser, leads, and commutator, and (2) for leads and commutator without the condenser. Since one-tenth of an ohm is the smallest coil in arm  $\alpha$  it is usually impossible to get an exact balance at this particular speed. Having obtained the chronograph run, the resistance of  $\alpha$  is made  $\alpha_1$  and the observer at the main bridge slowly alters the resistance in the auxiliary bridge, thus causing the auxiliary observer to slowly change the speed of the machine so as to maintain a balance in his bridge; the main observer continues this process while continually watching his galvanometer, until the main bridge is also balanced. Thus a simultaneous balance is obtained, and the amount by which the resistance in the auxiliary bridge has been altered, together with the former chronograph run, gives us directly the speed at which the main bridge is balanced when  $\alpha$  has the value of  $\alpha_1$ . Connections are now changed so as to measure the capacity of the leads and commutator only,  $\alpha$  is made equal to  $\alpha_2$ , and the process is repeated. By this process a balance can be obtained in forty to sixty seconds, while two and a half or three minutes is necessary to obtain a chronograph record of suitable length; as a consequence we can pass much more rapidly from the measurement of the capacity of the condenser, commutator, and leads to the measurement of the capacity of the commutator and leads only, and back again. This reduces the chance of the commutator capacity differing appreciable in the two cases, and enables us to make in rapid succession more determinations of the capacity in each case, both of which are conducive to greater accuracy.

This method presupposes, of course, that the capacity in the auxiliary bridge remains constant, that all variations in the resistances in this bridge are known, and that the arm in which the resistance is varied has a resistance sufficiently great to give the required accuracy of setting.

If the condenser is a good mica condenser, and if its temperature is maintained constant, which can readily be done, its capacity will remain constant to the highest degree of accuracy attainable in the measurement of capacities. Added to this, however, we have the capacity of the leads and of the commutator. If the capacity of the condenser is sufficiently large, as is always the case, the variations of these small capacities will be absolutely negligible as compared with the total capacity, unless the commutator is acting very badly. This last condition is, however, easily detected and corrected.

The only thing that can cause unknown variations in the resistances during the course of a day's work is the heating by the current, which is made rather heavy in order to obtain the desired sensibility. This effect, however, is reduced by a sufficient subdivision of the higher resistances and by closing the battery circuit a little while before beginning observations. The remainder of this effect, as well as all other slight variations, may be eliminated by taking a suitable number of chronograph runs at intervals throughout the day's work.

The total resistance in the arm of the auxiliary bridge which was varied was 50,000 ohms, and usually any one setting could be determined to within 0.5 ohm when the air condenser was in the main bridge, and to 1 ohm when the commutator and leads only were being measured. The mean of from three to six independent settings was always taken for each determination of the capacity.

In order to avoid all personal bias, the resistance which was varied was divided into two parts; one was placed near the auxiliary observer and the other near the main observer. After each setting the auxiliary observer changed his portion of the resistance by an amount unknown to the main observer, who then proceeded to change his portion of the resistance so as to restore the balance. In this way every setting was independent of the preceding one.

As it is seldom that both galvanometers are simultaneously at rest, it is often desirable for the auxiliary observer to call out,

at intervals of about one second, whether the speed is fast, slow, or right, according to the indications of his galvanometer. This serves as a guide to the main observer in estimating the correctness of the balance of his bridge. If the galvanometer in the main bridge has a period that is considerably longer than that of the one in the auxiliary bridge it will integrate the current more perfectly and the process of balancing will be rendered easier.

#### 47. THE NEW DIRECT READING CHRONOGRAPH.

In the earlier part of our work we used a chronograph<sup>19</sup> driven electrically and making one revolution of the drum per minute. By means of an electric contact on the shaft of the commutator every fiftieth revolution of the commutator was recorded on the chronograph, and a simultaneous record of the second-beats of a chronometer gave a means of computing with great precision the mean speed of the motor and commutator for any three-minute interval. The great labor of reading the chronograph record and computing the speed, together with the impossibility of detecting small variations in the speed until after the record had been worked up, led us to design and build a new form of chronograph which has been of great value in this and other work where the speed of a machine or the frequency of a current must be known with high precision.

The chronograph is shown in Fig. 39, and the gears and connections to the driving motor are shown in Fig. 40. The chronograph is driven through the worm and bevel gears at a speed 250 times less than the motor, which also drives the commutator and an alternating generator which provides testing current for other purposes, the frequency of which current has sometimes to be determined. The usual speed of the machine was 1,500 revolutions per minute, and the drum of the chronograph therefore made 6 revolutions per minute, or one in ten seconds. This speed could, however, be varied through wide limits, in practice from 500 to 2,000 or more per minute.

A printing magnet<sup>20</sup> was used instead of a pen, and once every second it printed (through a typewriter ribbon supported on posts  $P_1$  and  $P_2$ ) a dot on the paper on the drum D of the chronograph.

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<sup>19</sup> By the Société Genevoise.

<sup>20</sup> Such as used on the Rosa curve tracer.

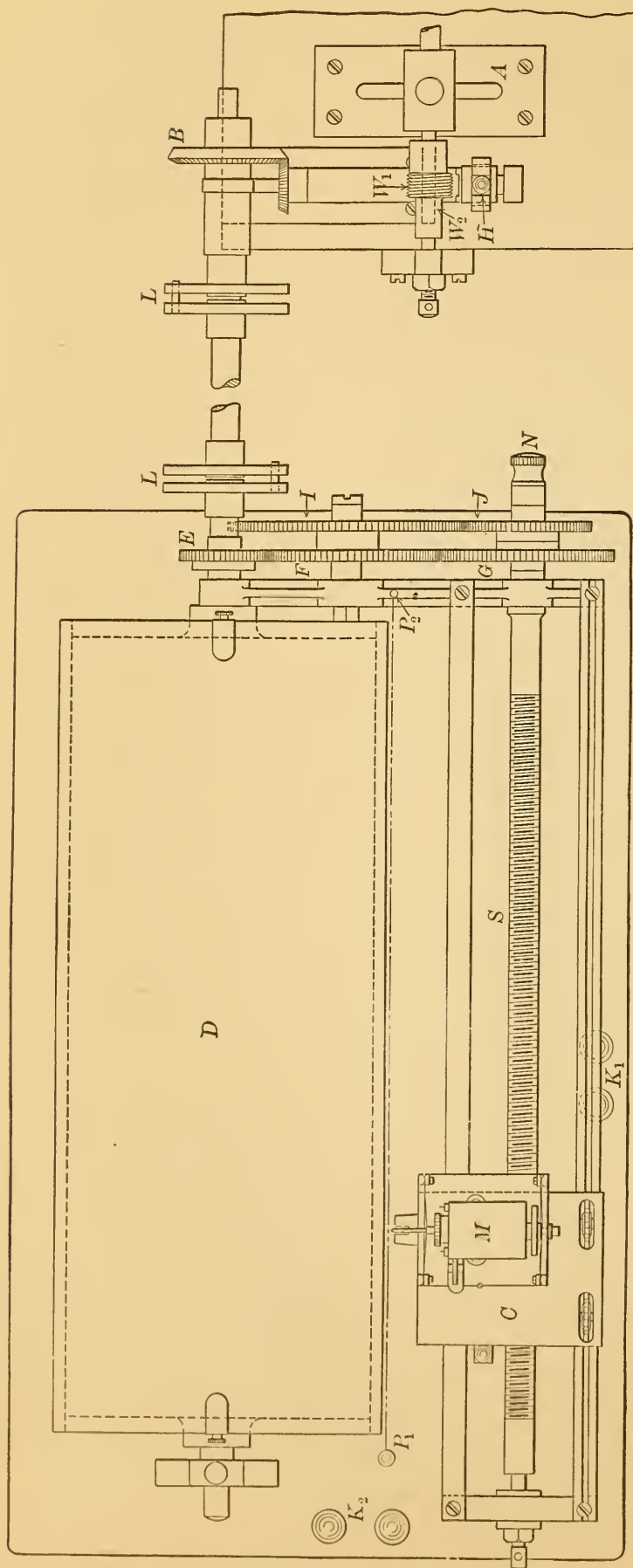


Fig. 40.—Direct Reading Chronograph for Giving the Speed of a Generator, Commutator, or Other Machine.

The chronograph is driven by the machine, the speed of which is to be measured, by means of the worm and gear  $W_1$  and  $W_2$  and the flexible coupling  $L$ . The bevel gears  $B$  are used merely to keep the direction of the axis of the chronograph parallel to that of the driving machine. The clutch  $N$  permits the speed of the driving screws to be changed. The printing magnet  $M$  (mounted on the carriage  $C$ ) prints through the type ribbon  $R$  upon the paper wrapped over the drum  $D$ . The binding posts  $K_1$  are joined by flexible wires to the printing magnet, and the binding posts  $K_2$  (connected to  $K_1$  in the base) are connected to the battery and chronometer. A dot is printed once a second on the revolving drum.

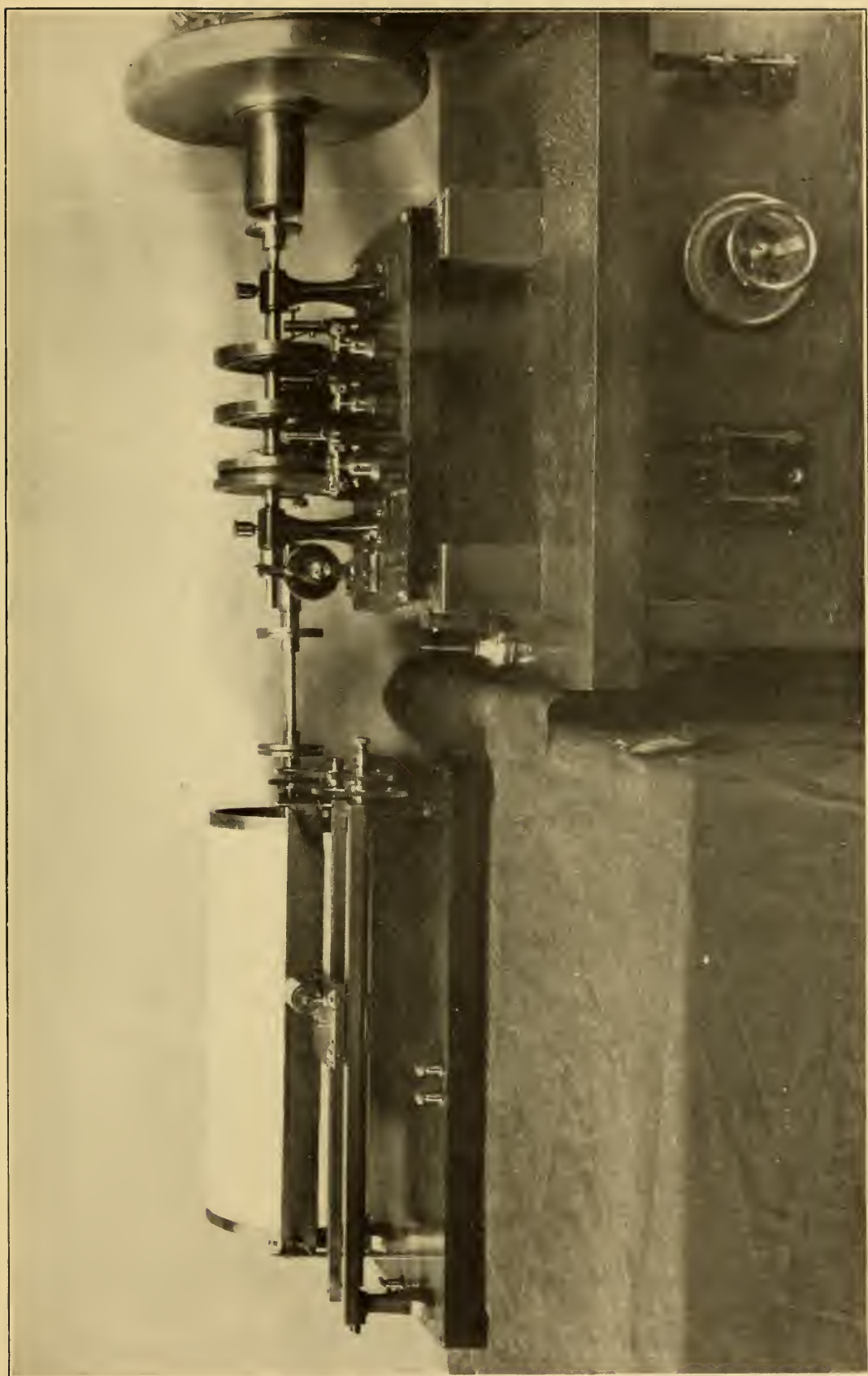


Fig. 39.—Direct Reading Chronograph.



At a speed of exactly 1,500 revolutions per minute there would be 10 dots in each revolution of the drum. The printing magnet was advanced by a carriage C and screw S like the pen of an ordinary chronograph, and hence there are 10 rows of dots printed, at the rate of 6 dots per minute in each row. If the speed of the motor is 1,250 instead of 1,500 there will be 12 rows of dots, printed at the rate of 5 per minute in each row, etc. The following are some of the different speeds that may be used to obtain horizontal rows of dots:

TABLE XV.

| Speed              | Number of rows | Number of dots per minute in each row |
|--------------------|----------------|---------------------------------------|
| 2,500              | 6              | 10                                    |
| 1,875              | 8              | $7\frac{1}{2}$                        |
| $1,666\frac{2}{3}$ | 9              | $6\frac{2}{3}$                        |
| 1,500              | 10             | 6                                     |
| 1,250              | 12             | 5                                     |
| 1,000              | 15             | 4                                     |
| 750                | 20             | 3                                     |
| 500                | 30             | 2                                     |

If the speed is a little greater or less than 1,500, for example, the rows of dots will slope one way or the other, the angular deviation from the horizontal being a measure of the variation of the speed from the nominal speed, the direction of the slope showing whether the speed is too fast or too slow. If the speed is variable the line of dots will be wavy or irregular. One can therefore see at a glance whether the speed is regular, and soon learns to estimate the speed without stopping the chronograph (if it is steady and near the nominal value) to within a few parts in 10,000. The interval between successive rows of dots at a speed of 1,500 is 5 cm, and this corresponds to one second. Hence the record can be read much closer than the usual chronograph record.

If AB, Fig. 41, is a row of dots making an angle  $a$  with the horizontal line AC, the speed was slower than the nominal speed in that case by an amount proportional to  $\tan a$ . The direction of rotation of the drum is indicated by the arrow.

The drum is 50 cm in circumference; hence if one revolution is

made in  $n$  seconds, the surface speed is  $50/n$  cm per second. The distance  $d=BC$  therefore represents a lag of the cylinder of  $\Delta t = \frac{dn}{50}$  sec. Let  $t$  be the time in which the given number of revolutions have been made. As the carriage advances 1 mm with every revolution of the drum

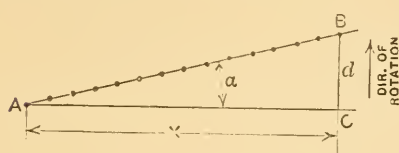


Fig. 41.

$$x = \frac{t}{10n} \text{ cm, or, } t = 10xn \text{ seconds}$$

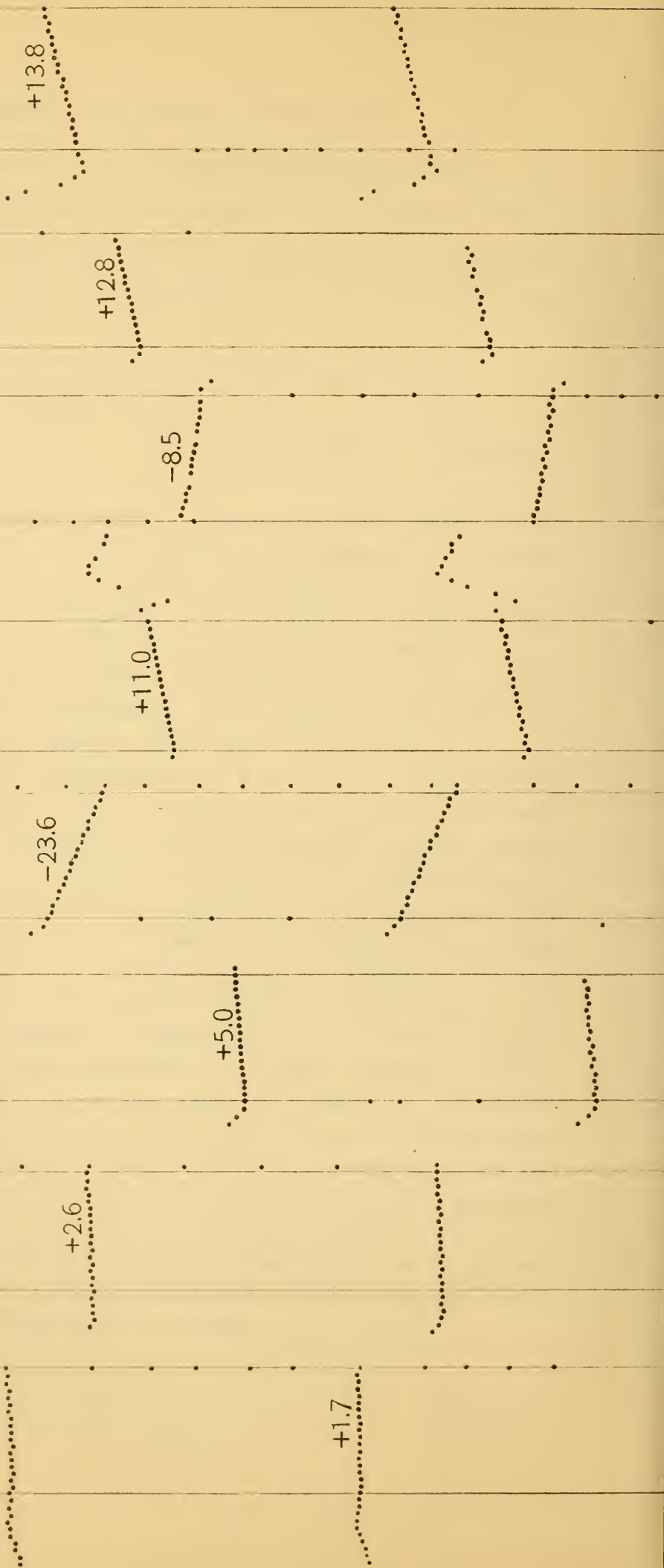
$$\therefore \frac{\Delta t}{t} = \frac{1}{500} \frac{d}{x} = \frac{2 \tan a}{1000} = \text{correction to}$$

be applied to the nominal speed to obtain the actual speed for the given interval. That is, *the correction in thousandths is twice the tangent of  $a$* . For  $a = 1^\circ$ ,  $2 \tan a = 0.035$ , and the correction is 3.5 in 100,000. As several independent rows of dots are available for the determination of  $a$ , the error in  $a$  need not be as much as one-fourth degree when the speed is steady for a run of two or three minutes. Hence the uncertainty in the speed is less than 1 in 100,000. For longer runs the uncertainty may be still less.

Fig. 42 is a full-size reproduction of a portion of a chronograph record taken October 9, 1906. It shows 8 runs of about three minutes each, between which are records of somewhat variable speeds. At the top of the sheet is given for each run the mean amount, in parts per thousand, by which the actual speed of the machine falls short of 1,500 revolutions per minute. As explained above, these corrections are equal to the doubled tangents of the angles of slope. For example, the first run on the sheet has a mean slope of  $1.8^\circ$  and the correction amounts to 6 parts in 100,000. The next figure (Fig. 43) is a reduced representation of an entire chronograph sheet, which was made in comparing some condensers by alternating current, the frequency being 100 at a speed of 1,500 of the machine. The corrected frequencies for the different runs are given at the top of the sheet. In this reproduction the tails, frequently produced when the duration of contact at the relay is excessive, have been unduly magnified, and the location of the points have not been faithfully reproduced. Further than this, it gives a fair idea of the general appearance of the sheets as taken.



+ .06      + .08      + .17      - .88      + .40      - .31      + .47      + .49



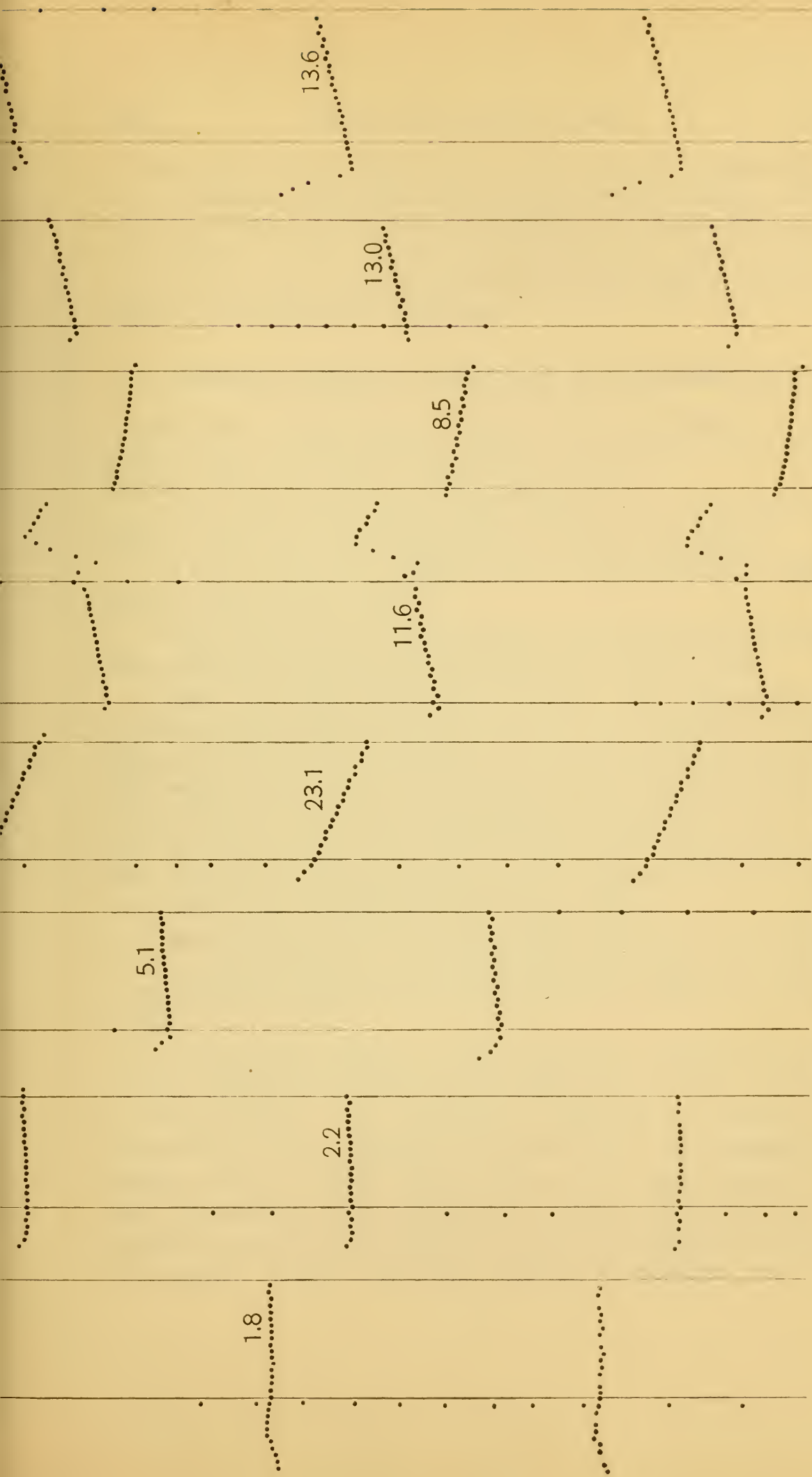


Fig. 42.—Portion of a Chronograph Record of October 9, 1906. Full size.

In the last run the upward slope indicates that the speed was slightly slower than the nominal speed, and the capacity being measured therefore a little greater than it would have been if the bridge had been balanced at the nominal speed. The corrections to the capacity are therefore positive when the speed is slower, and negative when faster than the nominal. The numbers on the rows of dots are the angular slopes in degrees as measured by a protractor, the mean corrections to the capacities are given at the top. The correction for the last run is 49 parts in 100,000.



The separate values of the slopes are given as read, a greater number of readings being taken in some cases than in others. The advantage of knowing the speed and whether it is steady while doing the experiment, and of getting its exact value in a moment by the use of a protractor will be appreciated by anyone who has used an ordinary chronograph for such work.

#### 48. POSSIBLE SOURCES OF ERROR DUE TO THE COMMUTATOR.

Aside from the question of leakage which has already been considered, there are five possible sources of error that may be introduced by the commutator. They are as follows:

1. The commutator increases the capacity to be measured by an amount which must be determined by a separate experiment.
2. If it cuts through a magnetic field, it may introduce a slight spurious electromotive force.
3. The friction between the segments and the brushes may introduce a differential thermal and frictional electromotive force.
4. Since the ebonite disc and the ebonite posts supporting the brushes become charged electrostatically when the commutator is running, the commutator will act like an electrostatic induction machine. Each segment (and therefore the condenser also) will acquire a certain charge, even when the battery is entirely disconnected, when the segment touches one brush, and will give it up when it touches the next. This charge adds itself algebraically to the charge given by the battery, and, if uncorrected, will lead to false results.

5. Electrostatic action of the commutator produced by the charging of the metallic brushes by the battery when the latter is in circuit.

Excepting the fourth, these sources of error occasion no serious trouble.

(1) Provided the capacity of the commutator remains constant or varies uniformly at a slow rate the first source of error occasions no inconvenience. It is necessary to eliminate the capacity of the leads from the condenser in any case, and if the commutator capacity, which is simply added to that of the leads, remains constant the capacity of the condenser (and charging wire in the case of the spheres) can be at once obtained by taking the difference between the capacity of the commutator, leads, and condenser and the

capacity of the commutator and leads alone. If the capacity of the commutator is varying slowly and uniformly, three capacity measurements only are needed, two of the capacity of the commutator and leads and one midway between the other two of the capacity of the commutator, leads, and condenser. One or other of these conditions can always be obtained by carefully adjusting the commutator brushes.

(2) In order that the second error shall be appreciable the magnetic fields normal to the two radii on which the brushes make contact must be very different. Such was not the case, however, in our experiment. The disk of the commutator lies in the plane of the earth's field and the stray field from the motor is very slight. Any slight effect of this kind that might exist would be eliminated along with the elimination of the electrostatic effect considered under the fourth head.

(3) The only condition under which the third source of error can affect the results is when the two brushes are unequally heated or make contact for unequal times. Such differences are very slight. Furthermore, the effect would be greatly enhanced by making the brushes of different materials. This was tested by making one brush of copper and the other of phosphor-bronze. The interchanging of these brushes produced no observable effect, thus showing that this source of error is so small as to be entirely masked by the next source of error to be considered. Furthermore, a small effect from this source will be eliminated along with the errors to be next considered.

(4) The fourth source of error is far greater and more troublesome than all the others taken together. It shows itself by producing a deflection of the galvanometer when the battery is entirely disconnected and the commutator is running. The deflection increases with the speed of the commutator, and its magnitude can be greatly changed by rubbing the ebonite posts or the disk of the commutator. In magnitude the deflection may vary from zero to two or three centimeters and may be either positive or negative, depending upon the distribution of the charges. Rubbing the posts produces a deflection in one direction; rubbing the disk produces a deflection in the opposite direction. By passing a flame over the ebonite surfaces or by gently rubbing one or other of them



| RUN   | 15     | 16      | 17     | 18      | 19      | 20     | 21      | 22      | 23      | 24     | 25      | 26      | 27      | 28     | 29      |
|-------|--------|---------|--------|---------|---------|--------|---------|---------|---------|--------|---------|---------|---------|--------|---------|
| ANGLE | -1.7   | +0.6    | -2.3   | +0.9    | +1.7    | -1.6   | +0.2    | +0.4    | +2.0    | -0.6   | +0.3    | +10.7   | +3.1    | -5.0   | +2.2    |
| FREQ. | 99.994 | 100.002 | 99.992 | 100.003 | 100.006 | 99.994 | 100.001 | 100.001 | 100.007 | 99.998 | 100.001 | 100.038 | 100.011 | 99.983 | 100.008 |

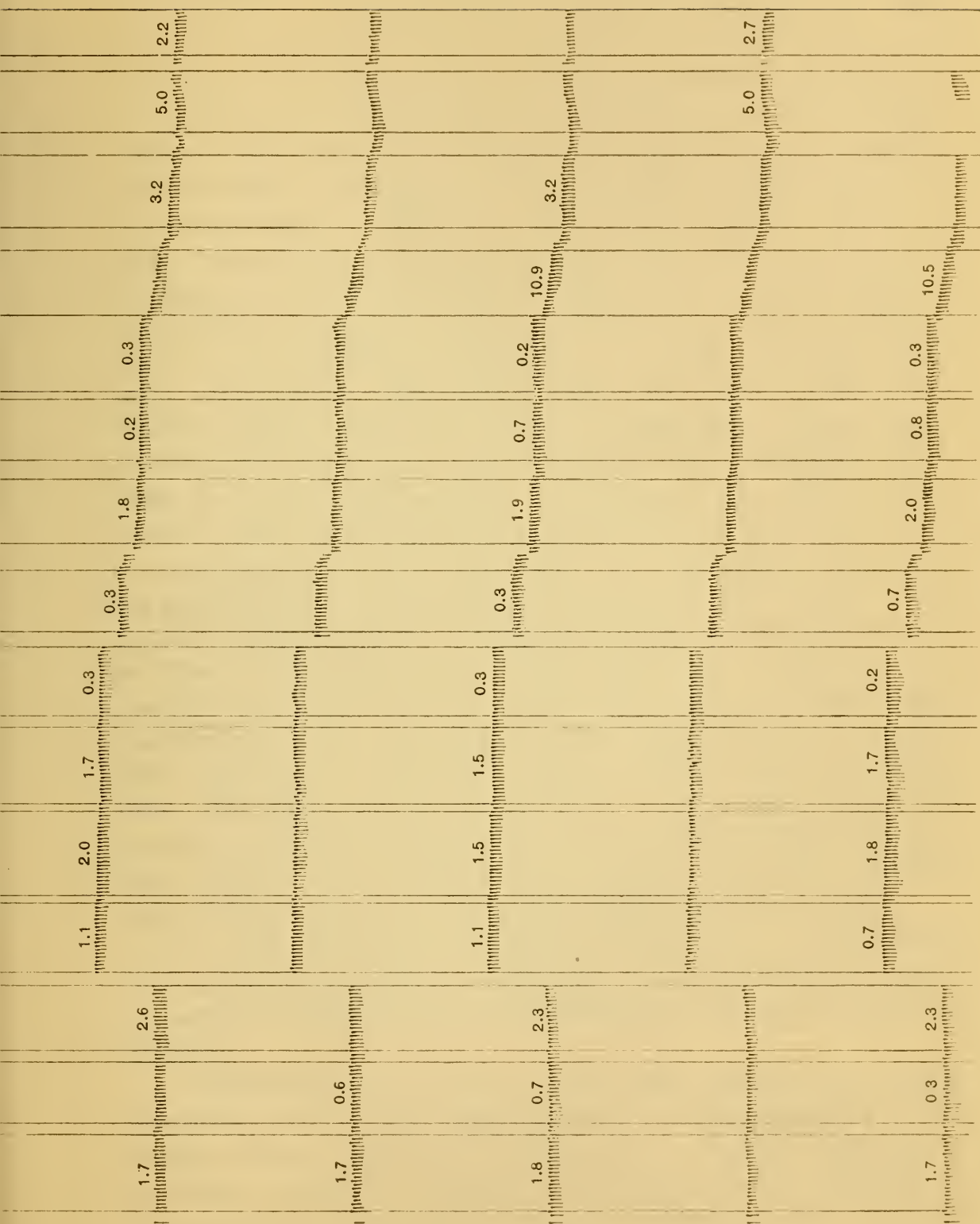


Fig. 43.—*Chronograph Record.*

This record was taken in comparing the capacity of a mica condenser with that of an air condenser, the actual frequency of the charging being given in the third line at the top. In this case the angle of an upward slope is marked negative, as it indicates a slower speed. The uncertainty of the measurement of these angles is indicated by the separate measurements on the different records of the same run. The variation of the separate measurements from the mean is usually less than 1 in 100,000.



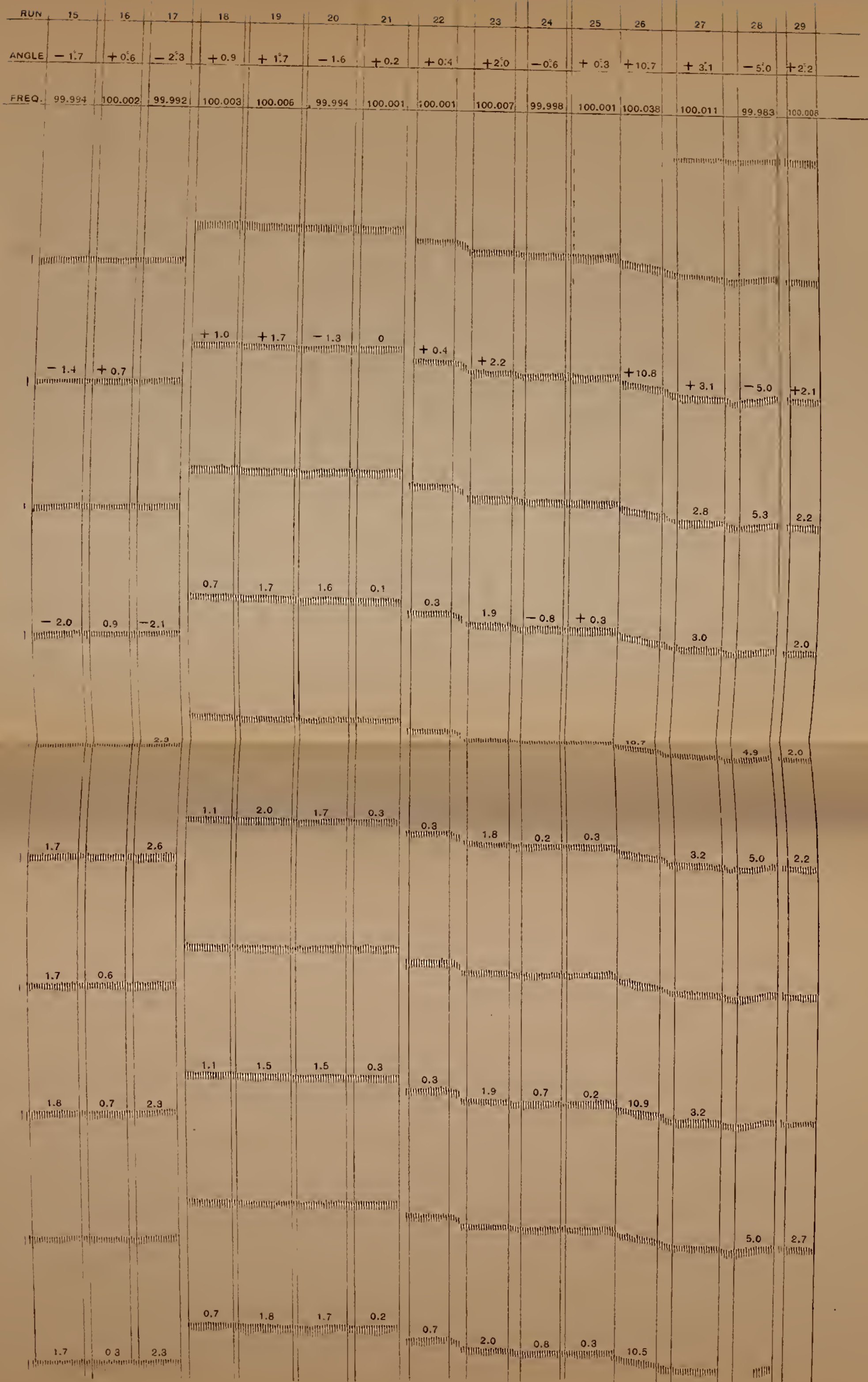


Fig. 43. —Chronograph Record.

This record was taken in comparing the capacity of a mica condenser with that of an air condenser, the actual frequency of the charging being given in the third line at the top. In this case the angle of an upward slope is marked negative, as it indicates a slower speed. The uncertainty of the measurement of these angles is indicated by the separate measurements on the different records of the same run. The variation of the separate measurements from the mean is usually less than 1 in 100,000.



the deflection can usually be reduced to a small value. A capacity determination was never attempted if the deflection exceeded 5 millimeters, and for most of the work it was under 2 millimeters. This deflection corresponds to the transfer of a definite quantity of electricity and is independent of the actual capacity being measured. Hence, since the capacity desired is the difference of two measured capacities, this effect will not affect the determination of the capacity desired unless the charges on the commutator are different for the two measurements. Therefore if this effect remains constant, or varies uniformly, it will be eliminated along with the capacity of the leads and of the commutator. However, we can not count on it remaining constant, or varying uniformly for any great length of time, so it is advisable to eliminate it as much as possible from every measurement. Two methods have been used to effect this. The first method employed consisted in measuring a capacity, then reversing the battery and the galvanometer and immediately measuring the same capacity again; if the mean of this effect was the same for both measurements, the mean of the two will be the value of the capacity freed from this effect. In this method the zero of the galvanometer was the zero on open galvanometer circuit. The second method consisted in taking for galvanometer zero its zero on closed circuit, the battery circuit being open. This zero is a spurious one, depending, as we have seen, upon the effect under consideration; hence measurements thus made will be free from this effect provided it remained constant during the observations. When the effect was very variable, the runs were made longer and the zero of the galvanometer was reset at intervals during the run. The variation in this effect was the greatest and most annoying source of error met with in the electromagnetic measurements.

It is evident that the errors mentioned under the second and the third groups also produce a deflection of the galvanometer when the battery is disconnected, and are eliminated by the methods just outlined.

(5) The fifth source of error is of the same nature as the fourth except that the inducing charges are upon the metal portions of the brushes and are maintained by the battery. Though always present when a capacity is measured by means of such a commutator, it can never be independently observed and allowed for. It takes

its rise in the fact that the commutator segment has a somewhat different capacity when it breaks contact with one brush from what it has when it breaks contact with the other. The difference in capacity is, however, independent of the capacity connected with the commutator; hence when the capacity is determined by the method of differences, as is always done, the final result is unaffected by this source of error.

#### 49. EFFECT OF VARIATIONS OF SPEED.

When considering the question of galvanometers, we arrived at the conclusion that even at the highest speed we ever used the duration of contact of the commutator brushes is ample to ensure completeness of the charge and of the discharge. Nevertheless it appeared desirable to test this point experimentally. For this purpose the difference between the capacity of the cylindrical condenser built up with three sections and no guard cylinder and the capacity of the same condenser when built up with but two sections was measured at different speeds, with the following results:

TABLE XVI.  
Effect of speed.

| Revolutions per minute | Charges per second | Capacity                   |
|------------------------|--------------------|----------------------------|
| 970                    | 259                | $76.570 \times 10^{-6}$ mf |
| 1,130                  | 301                | 76.558 "                   |
| 1,175                  | 313                | 76.556 "                   |
| 1,280                  | 341                | 76.555 "                   |
| 1,320                  | 352                | 76.556 "                   |
| 1,370                  | 365                | 76.556 "                   |
| 1,450                  | 387                | 76.556 "                   |
| 1,530                  | 408                | 76.575 "                   |
| 1,660                  | 443                | 76.560 "                   |
| 1,680                  | 448                | 76.559 "                   |
| 1,700                  | 453                | 76.560 "                   |
| 1,850                  | 493                | 76.555 "                   |
| 1,870                  | 499                | 76.579 "                   |
| Mean                   |                    | 76.561 "                   |

These results show that within this range the measured capacity is independent of the speed. When we consider that the capacity given above is the difference between two larger capacities, the greater of which was  $232 \times 10^{-6}$  mf the agreement is all that can be expected. The average variation from the mean is only  $0.006 \times 10^{-6}$  mf; that is, about one in 40,000 of the larger capacity, and there is no evidence that the value changes with the speed. The inner cylinder used in this work was not capped.

A few measurements of the capacity of ball B at different frequencies were made. These also indicate that the capacity as measured is independent of the frequency. The values found are given in Table XVII.

TABLE XVII.

Variations with speed. Ball B.

| Revolutions per minute | Charges per second | Capacity                   |
|------------------------|--------------------|----------------------------|
| 1,070                  | 288                | $32.979 \times 10^{-6}$ mf |
| 1,250                  | 333                | 32.981 "                   |
| 1,350                  | 360                | 32.986 "                   |
| 1,600                  | 427                | 32.977 "                   |
| 1,800                  | 480                | 32.983 "                   |
| Mean                   |                    | 32.981 "                   |

Here the average variation from the mean is only  $0.0026 \times 10^{-6}$  mf; that is, about one in 13,000 of the mean. When we remember that  $0.0026 \times 10^{-6}$  mf is the capacity of a sphere having a radius of but  $23 \mu$ , this variation is not astounding.

#### 50. EFFECT OF VARIATION OF VOLTAGE.

The battery, usually of about 200 volts, for charging the condenser consisted of small storage cells. This high voltage was chosen in order to reduce the relative importance of the electrostatic effect of the commutator. To test the effect of different voltages, a key was so connected as to allow the voltage to be suddenly changed between certain limits. Even when these limits were so different as 32 and 214 volts, the change produced no observable change in the

TABLE XVIII.  
Spherical Condenser, Ball B.

| Mean air temperature 25.8 C. Mean temperature of resistances 26.0 C for all. Hygrometer 62%. |                                 |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
|----------------------------------------------------------------------------------------------|---------------------------------|---------|-----------------------------|--------|----------------|----------------|--------|-------------------------------------|--------------------|-------------------------------------|--------------------|-------------------------------------------------------------------------------------------------|
| Run                                                                                          | Chr. time,<br>80,000<br>charges | Mean    | $\mu$<br>Nominal<br>Resist. | K      | K <sub>1</sub> | K <sub>2</sub> | k      | K - K <sub>1</sub> = c <sub>1</sub> | K <sub>2</sub> - k | K - K <sub>2</sub> = c <sub>2</sub> | K <sub>1</sub> - k | Remarks                                                                                         |
| 1                                                                                            | 179.550                         | 179.559 | 2162                        |        |                | 48.392         |        |                                     |                    |                                     |                    |                                                                                                 |
| 2                                                                                            | 179.568                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 3                                                                                            | 179.617                         | 179.600 | 2322                        | 51.982 |                |                |        | 1.672                               |                    | 3.590                               |                    |                                                                                                 |
| 4                                                                                            | 179.583                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 5                                                                                            | 179.472                         | 179.460 | 2249                        |        | 50.310         |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 6                                                                                            | 179.449                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 7                                                                                            | 179.700                         | 179.719 | 613                         |        |                |                | 13.723 |                                     |                    |                                     | 36.582             |                                                                                                 |
| 8                                                                                            | 179.738                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 9                                                                                            | 179.434                         | 179.428 | 2249                        |        | 50.301         |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 10                                                                                           | 179.422                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 11                                                                                           | 179.556                         | 179.564 | 2322                        | 51.972 |                |                |        | 1.671                               |                    | 3.595                               |                    |                                                                                                 |
| 12                                                                                           | 179.573                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 13                                                                                           | 179.513                         | 179.506 | 2162                        |        |                | 48.377         |        |                                     | 34.662             |                                     |                    |                                                                                                 |
| 14                                                                                           | 179.498                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 15                                                                                           | 179.626                         | 179.614 | 613                         |        |                |                | 13.715 |                                     |                    |                                     |                    | { After run 16, stopped<br>the machine and ad-<br>justed its brushes and<br>commutator brushes. |
| 16                                                                                           | 179.602                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 17                                                                                           | 180.018                         | 179.971 | 613                         |        |                |                | 13.742 |                                     |                    |                                     |                    |                                                                                                 |
| 18                                                                                           | 179.924                         |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 19                                                                                           | 179.604                         | 179.604 | 2162                        |        |                | 48.404         |        |                                     | 34.662             |                                     |                    | { Very unsteady; run<br>worthless; stopped for<br>lunch.                                        |
| 20                                                                                           |                                 |         |                             |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 21                                                                                           | 179.474                         | 179.461 | 2162                        |        |                |                |        |                                     |                    |                                     |                    |                                                                                                 |
| 22                                                                                           | 179.449                         |         |                             |        |                | 48.365         |        |                                     |                    |                                     |                    |                                                                                                 |

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|--------------------------------------|---------|---------|-------|--------|--------|--------|-----------|-------|--------|--------|-----------------------------------------------------------------------------------|
| 23                                   | 179.517 | 179.509 | 2322  | 51.956 | .....  | .....  | 1.676     | ..... | 3.591  | .....  | Balance unsteady. Af-<br>ter 28, stopped the<br>machine and fixed the<br>brushes. |
| 24                                   | 179.501 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 25                                   | 179.365 | 179.354 | 2249  | .....  | 50.280 | .....  | .....     | ..... | .....  | 36.580 |                                                                                   |
| 26                                   | 179.343 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 27                                   | 179.450 | 179.421 | 613   | .....  | .....  | 13.700 | .....     | ..... | .....  | .....  |                                                                                   |
| 28                                   | 179.392 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 29                                   | 179.489 | 179.476 | 2249  | .....  | 50.314 | .....  | .....     | ..... | .....  | 36.593 |                                                                                   |
| 30                                   | 179.463 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 31                                   | 179.720 | 179.692 | 613   | .....  | .....  | 13.721 | .....     | ..... | .....  | .....  |                                                                                   |
| 32                                   | 179.664 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 33                                   | 179.532 | 179.512 | 2162  | .....  | 48.379 | .....  | 34.661    | ..... | .....  | .....  |                                                                                   |
| 34                                   | 179.493 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 35                                   | 179.616 | 179.599 | 613   | .....  | .....  | 13.714 | .....     | ..... | .....  | .....  |                                                                                   |
| 36                                   | 179.582 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 37                                   | 179.513 | 179.503 | 2162  | .....  | 48.376 | .....  | 34.664    | ..... | .....  | .....  |                                                                                   |
| 38                                   | 179.494 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 39                                   | 179.560 | 179.555 | 613   | .....  | .....  | 13.710 | .....     | ..... | .....  | .....  |                                                                                   |
| 40                                   | 179.550 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 41                                   | 179.490 | 179.488 | 2162  | .....  | 48.372 | .....  | 34.664    | ..... | .....  | .....  |                                                                                   |
| 42                                   | 179.486 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| 43                                   | 179.552 | 179.513 | 613   | .....  | .....  | 13.707 | .....     | ..... | .....  | .....  |                                                                                   |
| 44                                   | 179.474 | .....   | ..... | .....  | .....  | .....  | .....     | ..... | .....  | .....  |                                                                                   |
| Capacity of charging wire.           |         |         |       |        |        |        | Mean..... | 1.673 | 34.663 | 3.592  | 36.584                                                                            |
| Capacity of Ball B.....              |         |         |       |        |        |        | .....     | ..... | 1.673  | .....  | 3.592                                                                             |
| Correction for resistance and time.. |         |         |       |        |        |        | .....     | ..... | .....  | .....  | .....                                                                             |
| Capacity of Ball B at 20°0 C.....    |         |         |       |        |        |        | .....     | ..... | .....  | .....  | .....                                                                             |
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balance of the bridge. Also for the plate condenser a few complete runs were made at different voltages. These likewise failed to indicate that the measured capacity depended in the least upon the voltage employed. Thus, the means of two runs at different voltages gave for the capacity of the condenser, leads, and commutator the following values:

At 32 volts,  $C = 137.481 \times 10^{-6}$  mf,

At 214 volts,  $C = 137.486 \times 10^{-6}$  mf.

A difference of only 5 parts in 137,000; the sensibility, and hence the accuracy with which the speed can be controlled, is of course much less at the lower voltage than at the higher one.

#### 51. METHODS FOR REDUCTION OF OBSERVATIONS.

Four different methods for reducing the observations were employed, depending upon the chronograph and the method employed in obtaining them.

In Table XVIII is shown the form of reduction employed in the earlier stages of the work. At this time we were using an electrically driven chronograph of a common form; the electrostatic effect of the commutator was eliminated by balancing to the galvanometer zero obtained from closed galvanometer but open battery circuit (see p. 567); after making a three-minute chronograph run the battery and galvanometer were both reversed and another run was made. The slight difference in the speed in the two cases is due to slight errors in the elimination of the electrostatic effect. The chronograph time for 80,000 charges is given in the second column of the table, and the mean of these times for each pair of runs taken as just described is given in the third column. This mean corrected to mean solar time (in this case a sidereal time chronometer was used) and combined with the assumed values of the resistances, give the capacities recorded in columns 5-8. The values assumed for the resistances are those based upon the most recent preceding comparison of the coils with the standards. As stated in section 44, these assumed values were later corrected for the seasonal changes in the resistances. This correction is combined with the correction for the temperature of the condenser and applied to the final result as given at the foot of the table. In the fourth column of the table

is given the nominal value of the resistance in the arm  $a$  of the bridge (see Fig. 29); the resistances in the other arms remained unchanged throughout the day. In the fifth column is given the electromagnetic capacity,  $K$ , of the commutator, lead wires, and ball with both charging wires in position. Removing the top charging wire,  $w_2$  (see Fig. 2) the electromagnetic capacity of the remaining portion is denoted by  $K_1$  and is given in column 6. Replacing  $w_2$  and removing the side charging wire,  $w_1$ , the capacity becomes  $K_2$  as given in the seventh column. Finally, removing both charging wires we obtain the capacity of the commutator and leads as given under  $k$  in column 8. The effective capacity of the upper charging wire,  $c_2$ , is the difference between  $K$  and  $K_1$ , that of the side wire,  $c_1$ , is  $K-K_2$ . The capacity of the ball increased by the effective capacity of the upper charging wire is equal to  $K_2-k$ ; the capacity of the ball increased by the capacity of the side charging wire is equal to  $K_1-k$  (see p. 444). These differences are given in the last four columns of the table. The unit in which all these capacities are expressed is the one-millionth of a microfarad.

In Table XIX is shown the method employed in reducing the results obtained by means of the differential galvanometer and the new form of chronograph.

Having decided upon a speed of 1,500 revolutions per minute (400 charges per second) as suitable for the work, a table was constructed giving the values of the capacity corresponding to given values of the resistance  $a$ . In this table (not given in this paper) the values of  $a$  (in the region with which we are concerned) differed by steps of one ohm each, so that it was easy to interpolate to a hundredth of an ohm in  $a$ . These capacities are of course based upon assumed values for the resistances of  $c$ ,  $d$ , and the galvanometer, and so are but approximately correct. From this table we get the values given in the third column of Table XIX. In the fourth column are given the corrections  $k_1$  (in parts per 1,000), which must be added to the approximate capacity on account of the speed of the machine being slightly different from that corresponding to exactly 1,500 revolutions per minute. As explained on page 564,  $k_1$  is merely twice the tangent of the angle of deviation of the rows of second dots on the chronograph sheet from the generating lines of the drum. In the fifth column is the sum of  $k_1$  and  $k_2$ , where  $k_2$  is the correction

TABLE XIX.

### Spherical Condenser, Ball B.—Differential Method.

Mean air temperature 21.0° C. Mean temperature of resistances 20.0° C for all. Hygrometer 50%.

November 24, 1905.

| Run | $a$<br>corrected | Approx.<br>capacity | $k_1$ | $k_1 + k_2$ | Total<br>correct. | k      | $K_2$  | $K_1$  | K      | $K - K_1$ | $K_2 - k$ | Mean   | Remarks     |
|-----|------------------|---------------------|-------|-------------|-------------------|--------|--------|--------|--------|-----------|-----------|--------|-------------|
| 1   | 2119.61          | 51.691              | +0.38 | +0.33       | +17               | .....  | .....  | .....  | 51.708 | .....     | .....     | .....  | Speed poor. |
| 2   | 2051.61          | 50.069              | +0.25 | +0.20       | +10               | .....  | .....  | 50.079 | .....  | .....     | .....     | .....  |             |
| 3   | 2120.61          | 51.714              | -0.56 | -0.61       | -32               | .....  | .....  | .....  | 51.682 | 1.604     | .....     | 1.610  |             |
| 4   | 2052.61          | 50.093              | -0.28 | -0.33       | -16               | .....  | .....  | 50.077 | .....  | 1.608     | .....     |        |             |
| 5   | 2119.61          | 51.691              | -0.01 | -0.06       | -3                | .....  | .....  | .....  | 51.688 | 1.612     | .....     |        |             |
| 6   | 2052.61          | 50.093              | -0.34 | -0.39       | -19               | .....  | .....  | 50.074 | .....  | 1.614     | .....     | .....  |             |
| 7   | 2119.61          | 51.691              | -0.01 | -0.06       | -3                | .....  | .....  | .....  | 51.688 | .....     | .....     | .....  |             |
| 8   | 546.51           | 13.557              | +1.98 | +1.93       | +26               | 13.583 | .....  | .....  | .....  | .....     | .....     | .....  |             |
| 9   | 547.51           | 13.581              | -0.17 | -0.22       | -3                | 13.578 | .....  | .....  | .....  | .....     | .....     | .....  |             |
| 10  | 1971.16          | 48.147              | +0.79 | +0.74       | +36               | .....  | 48.183 | .....  | .....  | .....     | 34.606    | .....  |             |
| 11  | 547.51           | 13.581              | -0.37 | -0.42       | -6                | 13.575 | .....  | .....  | .....  | .....     | .....     | .....  |             |
| 12  | 1972.16          | 48.171              | -0.19 | -0.24       | -12               | .....  | 48.159 | .....  | .....  | .....     | 34.584    | .....  |             |
| 13  | 547.51           | 13.581              | -0.39 | -0.44       | -6                | 13.575 | .....  | .....  | .....  | .....     | .....     | .....  |             |
| 14  | 1972.16          | 48.171              | -0.13 | -0.18       | -9                | .....  | 48.162 | .....  | .....  | .....     | 34.586    | .....  |             |
| 15  | 547.51           | 13.581              | -0.22 | -0.27       | -4                | 13.577 | .....  | .....  | .....  | .....     | .....     | .....  |             |
| 16  | 547.51           | 13.581              | -0.34 | -0.39       | -5                | 13.576 | .....  | .....  | .....  | .....     | .....     | 34.590 |             |
| 17  | 1972.16          | 48.171              | -0.19 | -0.24       | -12               | .....  | 48.159 | .....  | .....  | .....     | .....     | .....  |             |
| 18  | 1972.16          | 48.171              | -0.26 | -0.27       | -13               | .....  | 48.158 | .....  | .....  | .....     | 34.586    | .....  |             |
| 19  | 547.51           | 13.581              | -0.80 | -0.81       | -11               | 13.570 | .....  | .....  | .....  | .....     | .....     | .....  |             |



TABLE XX.

Spherical Condenser, Ball B.—Speed controlled by auxiliary bridge.

Mean air temp., 21.0° C. Mean temp. of resistances, 21.0° C.

November 12, 1906.

[illegible]



(in parts per 1,000) to be added to the capacity on account of the differences between the true values of the resistances of  $c$ ,  $d$ , and the galvanometer, and the values assumed in the construction of the table from which we obtain the approximate capacity as given in column 3. In the sixth column is given the correction  $k_1 + k_2$  expressed in thousandths of a micro-microfarad. The symbols at the heads of the following columns have the same meaning as in the former case.

As before, the values of the resistances assumed to be correct when the observations are reduced are slightly in error owing to the seasonal variations in the resistances. The correction for this error and the correction for the temperature of the condenser differing from  $20^\circ$  C are combined and applied to the final result, as shown at the bottom of the table.

The charging current was measured in the first 24 runs, and the discharging current was measured in the rest. These have been separately averaged and reduced, and give results differing by but 1 part in 16,000.

In order to test the equality of the coils of the galvanometer under working conditions four pairs of runs (17-18, 19-20, 21-22, 27-28) were taken, the second in each pair being taken immediately after the first, but with the galvanometer coils interchanged. No certain difference is observable.

As before, the electrostatic effect was eliminated by balancing to the displaced zero of the galvanometer.

In Table XX is shown the form used in the reduction of observations obtained by the method (page 558) in which the speed is controlled by means of an auxiliary bridge containing a large capacity. During this series four chronograph runs were taken, as shown in the table; the temperature of the large condenser was not kept perfectly constant so that the resistance  $c'$ , required to balance the auxiliary bridge when the speed of the machine was exactly 1,500 revolutions per minute gradually increased.

The first part of the table is taken up with the determination of the electromagnetic capacity of the upper charging wire ( $w_2$ ). "Wire in" denotes that this wire is in place so as to touch the ball and to dip into the mercury cup that serves to connect it with the commutator; "Wire out" denotes that it has been entirely removed. Throughout this portion of the work the side charging wire  $w_1$  was continuously in position so as to connect the ball to the commutator.

In column 2 is given the nominal value of the resistance in arm  $a$  of the main bridge; in the next four columns are given the values of the arm  $c'$  (decreased by 49,500 ohms) of the auxiliary bridge which correspond to a simultaneous balance of both bridges. The order in which these observations were taken is best illustrated by an example. Let us take the first set given in the table. The wire  $w_2$ , being in place,  $a$  is made equal to 2,137.5 ohms, and a simultaneous balance is found for  $c' = 49,599.0$  ohms; the auxiliary observer now changes the portion of the  $c'$  arm that is near him and the main observer (not knowing the amount of this change) proceeds to obtain another simultaneous balance of the bridges; this again occurs for  $c' = 49,599.0$  ohms. The wire is now taken out,  $a$  is made equal to 2,073.2 ohms, and two simultaneous balances are obtained, as before, for  $c' = 49,588.0$  and  $49,589.0$  ohms. Then the wire is replaced,  $a$  changed to its former value, and balances are obtained for  $c' = 49,599.0$  and  $49,597.0$  ohms, etc. The mean of these settings for each case is given in column 7. In the eighth column are given the values of  $c'_0 - 49,500$  that correspond to the balance of the auxiliary bridge for a speed of exactly 1,500 revolutions per minute; these are obtained by interpolating between the four chronograph runs taken during the day. The following column contains the difference between  $c'$  and  $c'_0$ , which (divided by  $c'_0$ ) gives the correction  $s$  (column 10) in parts per 1,000 to be added to  $a$  in order to obtain a balance for the even speed of 1,500 revolutions per minute. Column 11 contains the further correction to be added to  $a$  in order to compensate for the fact that  $cd$  has a value different from that assumed in the calculation of the capacity for a given value of  $a$ . The next column (headed  $Ap$ ) contains the correction that must be applied to the value of the capacity as calculated by the approximate formula  $\left(C = \frac{a}{ncd}\right)$  for the bridge in order to obtain the true capacity; the third and the fourth numbers from the end of this column are the corrections that must be applied on account of the galvanometer resistance being different from that assumed in the construction of the table giving the capacity corresponding to a given value of  $a$  when the differential method is used. The sum of these three corrections is given in column 13; the product of these sums by

one-thousandth of the corresponding  $\alpha$  is the correction in ohms to be applied to  $\alpha$ ; this is given in column 14. In column 15 is given the correction to be added to  $\alpha$  on account of the errors in the nominal values of the coils of which it is composed. The value of  $\alpha$  as given in column 2, increased by the corrections in columns 14 and 15, is exactly 40 times the true electromagnetic capacity, expressed in micro-microfarads, if the bridge method is used. If the differential method is used, the true capacity is that given by our table for this corrected value of  $\alpha$ . These capacities are given in column 16, and their differences (corresponding to the capacity of the wire or of the ball and wire) are given in columns 17 and 18.

The runs marked "Bridge" in the first column were obtained by the bridge method by the use of the Weston (low resistance) galvanometer; that marked "Charge" was by the differential method, the charging current being measured; and the last, marked "Bridge S," is by the bridge method, the Sullivan (high resistance) galvanometer being used.

The three methods substantially agree, and give  $32.967 \times 10^{-6}$  mf as the capacity of ball B at 20° C.

## 52. RESULTS BY BALL A.

Many preliminary determinations of the electromagnetic capacity of ball A were made, but all of those made before March 7, 1905, are unreliable on account of errors in the determination of the capacity of the charging wire, heating of the resistance in arm  $d$ , leakage, or other cause. On March 6, 1905, the 100,000 ohms resistance composing arm  $d$  was placed in oil. By this time the method for determining the capacity of the charging wire had been perfected, and the leakage had been reduced so as to be usually negligible. This date is therefore taken as the beginning of the actual measurements. However, there would occasionally be enough leakage over the ebonite bushing, which served as a guide for the side charging wire, to render the determination of the capacity of the charging wires and of the ball, when measured by means of the side charging wire, uncertain. This condition of affairs continued until April 4, 1905, when the bushing was replaced by one extending further beyond the surfaces of the shell and having the hole enlarged at each end so as to increase the leakage path. After this no further trouble from this cause was experienced.

Owing to this trouble some of the observations taken between March 7 and April 4 had to be discarded. All the determinations of the capacity of ball A, made after April 4, 1905 (except 6 single determinations out of a total of 65), are given in Table XXI. The observations omitted compose two groups of consecutive observations and every indication pointed to the fact that during these observations the commutator brushes were too loose.

TABLE XXI.  
Results by Ball A.

| Date | Wire | C     | $\delta_1$ | A  | $\delta_2$ | N  | W | v             | Mean    |
|------|------|-------|------------|----|------------|----|---|---------------|---------|
| 1905 |      |       |            |    |            |    |   |               |         |
| Mar. | 7    | 1.516 | 13.928     | 3  | 55.920     | 3  | 3 | $\frac{1}{2}$ | 2.9965  |
|      | 7    | 3.339 |            |    | 55.915     | 8  | 2 | $\frac{1}{2}$ | 2.9967  |
| Apr. | 10   | 1.454 | 14.026     | 3  | 55.922     | 2  | 3 | 3             | 2.9965  |
|      | 10   | 3.448 |            |    | 55.919     |    | 1 | 1             | 2.9966  |
|      | 17   | 1.454 | 13.883     | 2  | 55.929     | 1  | 2 | 2             | 2.9963  |
|      | 26   | 1.495 | 14.106     | 26 | 55.928     | 9  | 3 | 2             | 2.9963  |
|      | 26   | 3.484 |            |    | 55.932     |    | 1 | 1             | 2.9962  |
|      |      |       |            |    |            |    |   |               | 2.99614 |
| May  | 1    | 1.482 | 14.031     | 2  | 55.944     | 3  | 4 | 3             | 2.9959  |
|      | 1    | 3.433 |            |    | 55.944     | 4  | 4 | 3             | 2.9959  |
|      | 2    | 1.475 | 14.014     | 15 | 55.939     | 5  | 5 | 4             | 2.9960  |
|      | 2    | 3.429 |            |    | 55.939     | 6  | 3 | 2             | 2.9960  |
| July | 14   | 1.541 | 13.787     | 9  | 55.923     | 1  | 3 | 3             | 2.9964  |
|      | 15   | 1.504 | 13.741     | 15 | 55.962     | 2  | 2 |               |         |
|      | 15   | 3.476 |            |    | 55.929     | 4  | 2 | 1             | 2.9963  |
|      | 18   | 1.512 | 13.761     | 20 | 55.950     | 2  | 3 | 2             | 2.9959  |
|      | 18   | 3.471 |            |    | 55.934     | 12 | 2 | 1             | 2.9963  |
|      | 19   | 1.505 | 13.794     | 2  | 55.942     | 2  | 2 | 1             | 2.9961  |
|      | 19   | 3.472 |            |    | 55.944     | 2  | 2 | 1             | 2.9961  |
|      | 21   | 1.574 | 13.749     | 3  | 55.928     | 1  | 2 | 2             | 2.9963  |
|      | 21   | 3.466 |            |    | 55.928     | 2  | 2 | 2             | 2.9963  |
|      | 22   | 1.572 | 13.766     | 1  | 55.938     | 4  | 2 | 1             | 2.9960  |
|      | 22   | 3.472 |            |    | 55.934     | 6  | 2 | 1             | 2.9962  |
|      | 24   | 1.575 | 13.758     | 5  | 55.932     | 1  | 2 | 2             | 2.9962  |
|      | 24   | 3.474 |            |    | 55.932     | 2  | 2 | 2             | 2.9962  |
|      | 25   |       | 13.692     | 3  | 55.926     | 4  | 4 | 4             | 2.9964  |
|      | 25   |       |            |    | 55.920     |    | 1 | 1             | 2.9965  |

The only other result with this ball that does not appear in the final mean is the first given under date of July 15, 1905, in the table. Though everything appeared to be going nicely when these observations were taken, except that the humidity was high (70%), the results when reduced were found to be very erratic.

All the determinations with ball A were made by the bridge method.

The second column of the table contains the capacity of the charging wire; the smaller capacity (1.5) is that of the wire which passed through the hole in the top of the shell, the other (3.5) is that of the one which passed through the hole at the equator of the shell. In the third column under C is given the capacity of the commutator and leads; in the next column is given  $\delta_1$ , which is the average variation of the capacity of the commutator and leads from the mean value given under C. In the fifth column under A is given the capacity of ball A at 20°0 C (all corrections applied), and in the following column is given the average variation  $\delta_2$  of its measured capacity from this mean. N is the number of distinct determinations included in the mean value of the capacity given under A, and W is the weight assigned to this mean. In the ninth column under  $\sqrt{v}$  is the square root of the ratio of the electrostatic capacity of ball A (corrected for the holes in the shell, ebonite bushings, etc.) to the value given in column A.

The unit in which all the electromagnetic capacities are expressed is the one-millionth of a microfarad. The average variations from the mean ( $\delta_1$  and  $\delta_2$ ) are expressed in terms of units in the last place—that is, in terms of one-thousandth of a micro-microfarad.

It will be noticed that  $\delta_1$  is as a rule considerably greater than  $\delta_2$ . The cause of this is obvious. Every change in the adjustment of the brushes changes the value of the commutator capacity, and the brushes were usually readjusted several times in the course of a day. This sometimes caused the commutator capacity to vary over wide limits, and so made  $\delta_1$  large. But the value of the capacity of the ball is the difference between the measured capacity of the ball, commutator, and leads, and the mean of two measurements of the capacity of the commutator and leads, one being taken before and the other after the measurement with the ball. The commutator brushes are never touched during such a set of three measurements,

so this difference is independent of the large changes in the commutator capacity and is almost independent of the small ones.

The results naturally fall into four groups, which have been separately averaged. After each group the condenser was entirely dismantled and was reassembled again just before the beginning of the results recorded in the following group.

The mean value of  $\delta_2$  is but 0.0037 micro-microfarads, which shows that on the average a single measurement of the capacity of ball A departed from the mean of all measurements of the same day by no more than the capacity of a sphere of radius equal to 33 microns, or 7 parts in 100,000.

The weighted mean of these 64 independent determinations is

$$V = 2.99621 \times 10^{10}$$

The average variations of the results in the ninth column of the table from this mean is  $0.00018 \times 10^{10}$ , or 6 parts in 100,000.

### 53. RESULTS BY BALL B.

The preliminary work having been completed with ball A, ball B was adjusted in the shell on March 13, 1905, and the first determination of its electromagnetic capacity was made on the following day. As stated in the preceding section, we were occasionally troubled with leakage until April 4, 1905, so that some of the results in March had to be rejected. All the determinations of the capacity of ball B made after the latter date (except 3 single determinations out of a total of 202) are given in Table XXII. These omitted determinations were the last observations taken on November 21, 1905, when the commutator brushes were badly worn, and the commutator was working badly. There were in addition a few preliminary runs taken with the differential galvanometer. These were taken for the purpose of studying the instrument and the method and were not used for the purpose of obtaining a measurement of the capacity of the condenser.

Table XXII is analogous to Table XXI; the only change is that the column containing the capacity of the ball is headed B, and a column is inserted between this column and the column containing the mean deviations ( $\delta_2$ ). This column, which is marked M, con-

tains the designations of the methods employed; b, signifies that the Maxwell bridge method has been used; d, that the *discharge* has been measured by the differential galvanometer method; and c, that the *charge* has been measured by the same method. The results obtained on September 1 and 6, 1905, at different speeds have been averaged and recorded separately; the results for each day are arranged in the order of increasing speed. The measurements in October and November of 1906 were made by the method in which

TABLE XXII.

Results by Ball B.

| Date  | Wire | C     | $\delta_1$ | B  | M      | $\delta_2$ | N | W | v | Mean   |
|-------|------|-------|------------|----|--------|------------|---|---|---|--------|
| 1905  |      |       |            |    |        |            |   |   |   |        |
| Mar.  | 14   | 1.601 | 13.935     | 2  | 32.984 | b          | 0 | 3 | 3 | 2.9962 |
|       | 14   | 3.464 |            |    | 32.980 | b          | 2 | 3 | 3 | 2.9964 |
|       | 16   | 1.603 | 13.932     | 4  | 32.984 | b          |   | 1 | 1 | 2.9962 |
|       | 18   | 1.609 | 13.959     | 1  | 32.984 | b          |   | 1 | 1 | 2.9962 |
|       | 21   | 1.616 | 13.996     |    | 32.977 | b          |   | 1 | 1 | 2.9965 |
|       | 22   | 1.627 | 13.965     | 3  | 32.980 | b          | 2 | 5 | 5 | 2.9964 |
|       | 22   | 3.449 |            |    | 32.977 | b          | 4 | 3 | 3 | 2.9965 |
|       | 24   | 1.621 | 13.975     | 4  | 32.984 | b          | 2 | 3 | 3 | 2.9962 |
|       | 24   | 3.440 |            |    | 32.983 | b          | 2 | 3 | 3 | 2.9962 |
|       | 25   | 1.632 | 13.962     | 2  | 32.981 | b          | 2 | 3 | 3 | 2.9963 |
|       | 25   | 1.632 | 13.984     | 1  | 32.980 | b          | 1 | 3 | 3 | 2.9964 |
| July  | 26   | 1.656 | 13.675     | 1  | 32.978 | b          | 1 | 2 | 2 | 2.9965 |
|       | 26   | 3.584 |            |    | 32.979 | b          | 2 | 2 | 2 | 2.9964 |
|       | 27   | 1.662 | 13.683     | 5  | 32.982 | b          | 8 | 2 | 2 | 2.9963 |
|       | 27   | 3.582 |            |    | 32.985 | b          | 2 | 2 | 2 | 2.9962 |
|       | 28   | 1.666 | 13.691     | 7  | 32.976 | b          | 1 | 4 | 3 | 2.9965 |
|       | 28   | 3.587 |            |    | 32.981 | b          | 4 | 4 | 4 | 2.9963 |
|       | 31   | 1.673 | 13.716     | 9  | 32.980 | b          | 1 | 3 | 3 | 2.9964 |
|       | 31   | 3.592 |            |    | 32.982 | b          | 5 | 5 | 5 | 2.9963 |
| Sept. | 1    | 1.672 | 13.814     | 11 | 32.977 | b          | 3 | 5 | 5 | 2.9965 |
|       | 1    |       | 13.797     | 1  | 32.979 | b          | 3 | 2 | 2 | 2.9964 |
|       | 1    |       | 13.756     | 1  | 32.974 | b          | 3 | 2 | 2 | 2.9967 |
|       | 6    | 1.666 | 13.786     | 1  | 32.984 | b          | 2 | 2 | 2 | 2.9962 |
|       | 6    |       | 13.764     | 8  | 32.976 | b          | 4 | 4 | 4 | 2.9965 |
|       | 6    |       | 13.753     | 1  | 32.981 | b          | 1 | 2 | 2 | 2.9962 |

TABLE XXII—Continued.  
Results by Ball B—Continued.

| Date  | Wire | C     | $\delta_1$ | B  | M      | $\delta_2$ | N | W  | v  | Mean   |
|-------|------|-------|------------|----|--------|------------|---|----|----|--------|
| 1905. |      |       |            |    |        |            |   |    |    |        |
| Oct.  | 30   | 1.623 | 13.507     | 1  | 32.985 | d          | 3 | 2  | 2  | 2.9962 |
|       | 31   | 1.645 | 13.532     | 0  | 32.975 | b          | 3 | 2  | 2  | 2.9966 |
|       | 31   |       | 13.541     | 1  | 32.980 | d          | 1 | 3  | 3  | 2.9964 |
|       | 31   |       | 13.557     | 4  | 32.979 | c          | 1 | 3  | 3  | 2.9964 |
| Nov.  | 2    |       | 13.460     | 8  | 32.982 | b          | 2 | 4  | 4  | 2.9963 |
|       | 3    | 1.625 | 13.480     | 8  | 32.983 | b          | 5 | 3  | 2  | 2.9962 |
|       | 3    |       | 13.474     | 7  | 32.980 | c          | 1 | 3  | 3  | 2.9964 |
|       | 3    |       | 13.528     | 4  | 32.980 | d          | 4 | 3  | 3  | 2.9964 |
|       | 6    | 1.635 | 13.549     | 3  | 32.974 | d          | 3 | 4  | 4  | 2.9966 |
|       | 6    |       | 13.548     | 2  | 32.979 | c          | 6 | 4  | 4  | 2.9964 |
|       | 6    |       | 13.543     | 3  | 32.975 | b          | 3 | 4  | 4  | 2.9966 |
|       | 9    | 1.632 | 13.452     |    | 32.980 | d          |   | 1  | 1  | 2.9963 |
|       | 10   | 1.625 | 13.429     | 4  | 32.976 | d          | 1 | 2  | 1  | 2.9965 |
|       | 18   | 1.615 | 13.721     |    | 32.982 | d          |   | 1  | 1  | 2.9963 |
|       | 20   | 1.618 | 13.569     | 4  | 32.982 | d          | 1 | 3  | 3  | 2.9963 |
|       | 20   |       | 13.564     | 3  | 32.982 | c          | 3 | 3  | 3  | 2.9963 |
|       | 23   | 1.611 | 13.544     | 3  | 32.986 | b          | 5 | 4  | 4  | 2.9961 |
|       | 23   |       | 13.550     | 3  | 32.998 | d          | 6 | 2  | 2  | 2.9956 |
|       | 23   |       | 13.545     | 4  | 32.980 | c          | 6 | 8  | 7  | 2.9964 |
|       | 24   | 1.610 | 13.576     | 2  | 32.978 | c          | 6 | 4  | 4  | 2.9964 |
|       | 24   |       | 13.568     | 2  | 32.976 | d          | 2 | 3  | 3  | 2.9965 |
|       | 25   | 1.607 | 13.600     | 1  | 32.979 | d          | 1 | 2  | 2  | 2.9964 |
|       | 25   |       | 13.632     | 22 | 32.979 | c          | 4 | 4  | 4  | 2.9964 |
|       | 29   | 1.619 | 13.702     | 11 | 32.977 | b          | 2 | 5  | 5  | 2.9965 |
| 1906. |      |       |            |    |        |            |   |    |    |        |
| Oct.  | 30   | 1.605 | 15.393     | 1  | 32.975 | c          | 0 | 2  | 2  | 2.9966 |
|       | 30   |       | 15.410     | 2  | 32.977 | c          | 1 | 8  | 8  | 2.9965 |
|       | 30   |       | 15.399     | 1  | 32.978 | c          | 0 | 3  | 3  | 2.9965 |
|       | 30   |       | 15.415     | 1  | 32.974 | c          | 0 | 2  | 2  | 2.9966 |
| Nov.  | 1    | 1.604 | 15.411     | 2  | 32.971 | c          | 2 | 2  | 2  | 2.9967 |
|       | 1    |       | 15.407     | 2  | 32.974 | c          | 1 | 10 | 10 | 2.9966 |
|       | 12   | 1.613 | 15.337     | 1  | 32.967 | b          | 0 | 5  | 5  | 2.9969 |
|       | 12   |       | 15.335     | 0  | 32.966 | b          | 1 | 4  | 4  | 2.9970 |
|       | 12   |       | 15.333     | 2  | 32.970 | c          | 2 | 8  | 8  | 2.9968 |
|       | 12   |       | 15.328     | 0  | 32.966 | b          | 1 | 4  | 4  | 2.9969 |
|       | 15   | 1.609 | 15.154     | 0  | 32.981 | c          | 2 | 6  | 6  | 2.9963 |
|       | 16   | 1.607 | 15.381     | 1  | 32.977 | b          | 1 | 8  | 8  | 2.9965 |
|       | 16   |       | 15.381     | 1  | 32.980 | c          | 1 | 6  | 6  | 2.9964 |
|       | 16   |       | 15.385     | 1  | 32.977 | c          | 2 | 6  | 6  | 2.9965 |
|       | 16   |       | 15.390     | 2  | 32.980 | c          | 1 | 7  | 7  | 2.9964 |

the auxiliary bridge is employed (see p. 558). It will be noticed that  $\delta_1$  is here very much smaller than it is in the first part of the table; this is a consequence of the greater rapidity with which determinations can be made by this method; the commutator brushes are not touched at all during the determinations composing any one series.

These results fall into five groups, which have been separately averaged. As with ball A the condenser was entirely dismantled after each group and was reassembled again just before the beginning of the preceding group.

The mean value of  $\delta_2$  for the observations of 1905 is but 0.0030 micro-microfarads, or the capacity of a sphere of a radius of 27 microns. The mean value of  $v$  for this year is 2.99637. In October and November, 1906, when the other method was employed, the mean value of  $\delta_2$  was but 0.0011 micro-microfarads, or the capacity of a sphere of a radius of 10 microns. The mean value of  $v$  for this period is 2.99661. At the time these measurements were made we were aware that they departed from the mean, and every endeavor was made to locate the cause of the discrepancy. The resistances were carefully compared with the standards and also intercompared, all leakage paths were carefully tested, and on November 12 three methods were employed in determining the capacity. The first two results given under that date were determined by the bridge method, using the low-resistance galvanometer; the fourth was by the same method, but with the high resistance galvanometer, and the third was by the differential method, the charge being measured. The results by these three methods agree almost perfectly. The next day the guard cylinder condenser was assembled and one determination of the capacity was made. This value gave a remarkably low value for  $v$ . The next day several results were obtained that differed but little from the mean. The following days the results obtained by the cylinders gave rather low values for  $v$ , while those by the ball were rather high, but much lower than the values found on November 12. The cause of these high values has not yet been found. It may possibly be due to some abnormality in the dielectric constant of air during this period.

The weighted mean of the 147 independent determinations made in 1905 is

$$v = 2.99637 \times 10^{10}$$

The average deviation of the results in the tenth column of the table from this value is

$$0.00011 \times 10^{10}, \text{ or four parts in } 100,000.$$

The weighted mean of the 81 independent determinations made in 1906 is

$$v = 2.99659 \times 10^{10}$$

The average deviation determined as before is

$$0.00016 \times 10^{10}, \text{ or five parts in } 100,000.$$

From these results we can get a good comparison of the results obtained by the three methods designated as b, c, and d; collecting the results obtained by different methods on the same day we obtain the data presented in Table XXIII.

TABLE XXIII.

Comparison of Different Methods.

| Method.       | b        | c        | d        |
|---------------|----------|----------|----------|
| 1905.         |          |          |          |
| Oct. 31 ..... | 2.9966   | 2.9964   | 2.9964   |
| Nov. 3 .....  | 2.9962   | 2.9964   | 2.9964   |
| 6 .....       | 2.9966   | 2.9964   | 2.9966   |
| 23 .....      | * 2.9961 | * 2.9964 | * 2.9956 |
| Mean .....    | 2.99647  | 2.99640  | 2.99647  |
| Nov. 20 ..... |          | 2.9963   | 2.9963   |
| 24 .....      |          | 2.9965   | 2.9964   |
| 25 .....      |          | 2.9964   | 2.9964   |
| Mean .....    |          | 2.99640  | 2.99637  |
| 1906.         |          |          |          |
| Nov. 12 ..... | 2.9969   | 2.9968   | .....    |
| 16 .....      | 2.9965   | 2.9964   | .....    |
| Mean .....    | 2.99670  | 2.99660  | .....    |

\* Omitted from mean, balance hard to maintain, brushes bad.

## 54. RESULTS BY CYLINDERS WITHOUT GUARDS.

A series of measurements were made with the upper ends of both cylinders, and with the lower end of the outer one open. These are assembled in Table XXIV. The numbers at the tops of the columns indicate the two capacities of which the difference is given in that column; for example, 1. 2. 3-1. 2 at the head of the second column indicates that each number contained in this column is the

TABLE XXIV.

Difference in the Capacity produced by Changing the Height.

| Date.      | 1.2.3-1.2 | W | 1.2.3-1.3 | W | 1.3.2-1.3 | W | 1.3.2-1.2 | W | 1.2-1  | W | 1.3-1  | W |
|------------|-----------|---|-----------|---|-----------|---|-----------|---|--------|---|--------|---|
| 1905       |           |   |           |   |           |   |           |   |        |   |        |   |
| Mar. 24    | 76.542    | 1 |           |   |           |   |           |   | 76.152 | 1 |        |   |
| Apr 18     |           |   |           |   |           |   |           |   | 76.245 | 2 |        |   |
| 19         |           |   | 75.683(?) | 1 |           |   |           |   |        |   |        |   |
| 20         |           |   | 76.137    | 1 | 76.120    | 1 |           |   | 76.182 | 1 | 76.630 | 1 |
| 22         |           |   |           |   | 76.134    | 1 | 76.546    | 2 | 76.147 | 2 | 76.565 | 4 |
| May 2      |           |   | 76.172    | 1 |           |   |           |   | 76.204 | 2 |        |   |
| 3          | 76.589    | 2 |           |   | 76.143    | 2 | 76.606    | 2 | 76.195 | 2 |        |   |
| 8          | 76.582    | 1 |           |   | 76.121    | 1 |           |   | 76.185 | 2 | 76.605 | 1 |
| 9          | 76.558    | 2 | 76.078    | 2 |           |   | 76.590    | 2 | 17.178 | 3 |        |   |
| Aug. 1     | 76.564    | 6 |           |   | 76.132    | 4 |           |   | 76.174 | 5 | 76.618 | 4 |
| 2          | 76.553    | 4 | 76.133    | 4 | 76.129    | 4 |           |   | 76.175 | 4 | 76.627 | 4 |
| 3          | 76.553    | 3 |           |   |           |   |           |   |        |   |        |   |
|            | 76.577    | 3 |           |   |           |   |           |   |        |   |        |   |
|            | 76.558    | 2 |           |   |           |   |           |   |        |   |        |   |
| 23         | 76.565    | 2 |           |   |           |   |           |   |        |   |        |   |
|            | 76.555    | 2 |           |   |           |   |           |   |        |   |        |   |
| 24         | 76.580    | 3 |           |   |           |   |           |   |        |   |        |   |
|            | 76.578    | 2 |           |   |           |   |           |   |        |   |        |   |
|            | 76.573    | 1 |           |   |           |   |           |   |        |   |        |   |
| 25         | 76.564    | 2 |           |   |           |   |           |   |        |   |        |   |
|            | 76.523    | 1 |           |   |           |   |           |   |        |   |        |   |
|            | 76.519    | 1 |           |   |           |   |           |   |        |   |        |   |
| 31         | 76.554    | 4 |           |   |           |   |           |   |        |   |        |   |
|            | 76.551    | 4 |           |   |           |   |           |   |        |   |        |   |
|            | 76.554    | 2 |           |   |           |   |           |   |        |   |        |   |
| Sept. 7    | 76.563    | 4 |           |   |           |   |           |   |        |   |        |   |
|            | 76.562    | 3 |           |   |           |   |           |   |        |   |        |   |
|            | 76.546    | 3 |           |   |           |   |           |   |        |   |        |   |
| 12         | 76.552    | 3 |           |   |           |   |           |   |        |   |        |   |
| 13         | 76.558    | 3 |           |   |           |   |           |   |        |   |        |   |
| Mean       | 76.560    |   | 76.125    |   | 76.131    |   | 76.581    |   | 76.183 |   | 76.605 |   |
| Correction | -0.005    |   | -0.005    |   | -0.005    |   | -0.005    |   | -0.005 |   | -0.005 |   |
|            | 76.555    |   | 76.120    |   | 76.126    |   | 76.576    |   | 76.178 |   | 76.600 |   |

difference between the capacity of the condenser as composed of sections 1, 2, and 3 and its capacity as composed of sections 1 and 2. As before, the unit of capacity employed is one micro-microfarad.

While the individual values are rather variable, the means are probably fairly accurate and show that the first section added invariably produces a greater increase in the capacity of the system than the same cylinder produces when it is the second section added. This is exactly what we have found should be observed on account of the distribution of charge on the outside of the outer cylinder. It was not due to the change in position of the upper end with respect to objects above it, for the upper end never came closer than 80 cm to any of these objects, while by actual trial it was found that a metal plate could be brought down from above to within about 10 cm of the upper end of the condenser before it appreciably affected the balance of the bridge.

Taking the weighted means from Table XXIV we find the following values for the increase in the capacity produced by the addition of one section:

TABLE XXV.

| Section.            | 2      | 3      |
|---------------------|--------|--------|
| Section added first | 76.178 | 76.600 |
| “ “ second          | 76.124 | 76.557 |
| Difference          | 0.054  | 0.043  |

Assuming  $2.9963 \times 10^{10}$  as the true value of  $v$ , the electromagnetic capacities of these sections are 76.085 and 76.518 micro-microfarads, respectively. Hence the measured capacity is too great by  $124 \times 10^{-5}$  and  $107 \times 10^{-5}$  of the total for the section first added, and  $51 \times 10^{-5}$  of the total for the section next added. Hence the section first added has a capacity higher than that calculated by 12 parts in 10,000, and the next section is higher by 5 parts in 10,000. Hence in order to use this method the shortest section of the cylinder should be at least 70 or 80 cm in length. This, however, will in turn necessitate a great increase in the accuracy of the electromagnetic measurements, for the determination will then depend upon the dif-

ference of two large quantities. For this reason it is clearly not desirable to use this method with open cylinders.

**Capped cylinders.**—Recently a few measurements were made on two days with both cylinders capped at the top. As stated on page 522, it seems that with the arrangement adopted no error is introduced by the fact that the lower end of the outer cylinder is not capped. The results are as follows:

TABLE XXVI.

| Section.            | 2      | 3      |
|---------------------|--------|--------|
| First section added | 76.109 | 76.531 |
| Second " "          | 76.085 | 76.510 |
| Difference          | 0.024  | 0.021  |

Here we still have a difference in the same direction as before, but a little less than half as large. The values of the capacities are much nearer their true values than those obtained when the ends were open, but for the first section added they are still too large. The capacities for the sections added second give us  $\nu = 2.9963 \times 10^{10}$  for section 2, and  $2.9964 \times 10^{10}$  for section 3.

The only explanation of the high values found for the section added first seems to be that the capacity of the lower end of the cylinder, or of the leads and commutator, is appreciably affected by changing the height of the cylinder in spite of the tests described on page 522.

The method of procedure followed in this part of the work necessitates numerous buildups of the condenser, and the results obtained serve as an indication of the reproducibility of this process. From the table we see that the average variation from the mean for any section is only 0.010 micro-microfarads. This is very good when we consider that this includes the variations in the commutator capacity, and these may well be large, as it requires some time to add and adjust (or to remove) a section.

## 55. RESULTS BY CYLINDERS WITH GUARDS.

Having found by experience that it is impossible to obtain consistent results with uncapped cylinders without guard cylinders,

the guard cylinders were constructed and after a few preliminary experiments the determination of  $v$  by the use of the guard cylinder condenser was begun on March 8, 1906. The use of the second set of commutator brushes (for charging the guard cylinders) necessitated the setting of all the brushes at rather unfavorable angles so that the vibration of these brushes was increased. This in turn made the brushes work loose more rapidly, and rendered the capacity of the commutator more variable. Because of this some of the measurements between March 8 and March 16 had to be discarded, but after the latter date the adjustment of the brushes was maintained by frequently testing with a milammeter, as already described. As a consequence, all observations after March 16, 1906, are included in Table XXVII, except 7 single determinations (out of a total of 457) when there was a slight leakage across the mica insulation separating the segments of the ring on which the side brush bears, causing the capacity of the commutator to be uncertain. On January 3 and 4, 1907, the balance of the bridge could not be maintained, so that the observations made were discarded; they indicated a variable leakage, probably between the segments of the commutator.

The construction of Table XXVII is similar to that of Table XXI, except that in this case there is no charging wire of which the capacity has to be determined. All these determinations were made by the differential method, except those of October 8 and 9, 1906, which were by the bridge method. All the results after October 9 were obtained by the method involving the use of the auxiliary bridge (page 558).

The condenser was dismantled and reassembled again after each group that has been separately averaged.

The average value of  $\delta_2$  for the observations taken before November 14, 1906, is 0.0150 micro-microfarads; this is only 1 in 10,000 of the capacity of the condenser, but is absolutely about 5 times as large as in the case of the spheres. This increase is due to the additional difficulties introduced by the second set of commutator brushes. The weighted mean of these 174 independent determinations is

$$v = 2.99626 \times 10^1$$

TABLE XXVII.  
Results by Guard Cylinder Condenser.

| Date | C  | $\delta_1$ | Cylinder | $\delta_2$ | N     | W  | v             | Mean   |
|------|----|------------|----------|------------|-------|----|---------------|--------|
| 1906 |    |            |          |            |       |    |               |        |
| Mar. | 8  | 13.470     | 2        | 152.814    | 7     | 5  | 3             | 2.9964 |
|      | 8  | 13.483     | 2        | 152.841    | 3     | 6  | 3             | 2.9962 |
|      | 9  | 13.453     | 38       | 152.818    | 7     | 5  | 2             | 2.9964 |
|      | 9  | 13.477     | 5        | 152.891    | 5     | 2  | 1             | 2.9957 |
|      | 10 | 13.520     | 16       | 152.799    | 8     | 8  | 4             | 2.9966 |
|      | 10 | 13.449     | 50       | 152.773    | 76    | 6  | 3             | 2.9968 |
|      | 12 | 13.665     | 125      | 152.860    | 17    | 6  | 2             | 2.9960 |
|      | 14 | 13.488     | 3        | 152.881    | 6     | 4  | 2             | 2.9958 |
|      | 15 | 13.394     | 15       | 152.851    | 6     | 4  | 2             | 2.9961 |
|      | 16 | 13.937     | 119      | 152.839    | 8     | 8  | 4             | 2.9962 |
|      | 20 | 14.107     | 92       | 152.883    | 28    | 13 | 3             | 2.9958 |
|      | 24 | 13.832     | 68       | 152.779    | 7     | 10 | 5             | 2.9968 |
|      | 26 | 13.942     | 21       | 152.796    | 21    | 9  | 2             | 2.9966 |
|      | 26 | -----      | -----    | 152.838    | 7     | 6  | 3             | 2.9962 |
| Apr. | 28 | 13.905     | 49       | 152.825    | 4     | 4  | 2             | 2.9963 |
|      | 29 | 14.408     | 179      | 152.833    | 11    | 14 | 5             | 2.9962 |
|      | 30 | 14.902     | 29       | 152.782    | 18    | 12 | 4             | 2.9967 |
|      | 5  | 17.538     | 21       | 152.794    | 6     | 5  | 2             | 2.9966 |
|      | 5  | 17.560     | 1        | 152.859    | 6     | 5  | 2             | 2.9960 |
|      | 5  | 17.537     | 1        | 152.862    | 5     | 4  | 2             | 2.9960 |
|      | 7  | 17.982     | 5        | 152.827    | 9     | 4  | 2             | 2.9963 |
|      | 10 | 17.050     | 35       | 152.871    | 9     | 7  | 3             | 2.9959 |
|      | 15 | 20.836     | 3        | 152.904    | 13    | 3  | 1             | 2.9955 |
|      | 15 | 18.979     | 23       | 152.922    | 37    | 4  | 1             | 2.9954 |
| May  | 15 | 19.005     | 3        | 152.873    | 13    | 3  | 1             | 2.9959 |
|      | 15 | 19.005     | 2        | 152.866    | 2     | 3  | 3             | 2.9959 |
|      | 15 | 19.011     | 3        | 152.889    | 20    | 3  | 1             | 2.9957 |
|      | 17 | 19.062     | 4        | 152.805    | 13    | 3  | 1             | 2.9965 |
|      | 18 | 19.017     | 15       | 152.856    | 8     | 3  | 1             | 2.9960 |
|      | 8  | 20.325     | 11       | 152.770    | 5     | 2  | 1             | 2.9969 |
|      | 9  | 20.300     | 18       | 152.789    | 61    | 3  | $\frac{1}{2}$ | 2.9967 |
|      | 13 | 18.390     | -----    | 152.895    | ----- | 1  | $\frac{1}{2}$ | 2.9956 |
| Nov. | 14 | 18.354     | 1        | 152.825    | 7     | 6  | 3             | 2.9963 |
|      | 14 | 18.357     | 2        | 152.839    | 3     | 7  | 4             | 2.9962 |
|      | 14 | 18.360     | 1        | 152.829    | 2     | 6  | 3             | 2.9963 |
|      | 14 | 18.169     | 2        | 152.843    | 4     | 5  | 2             | 2.9962 |
|      |    |            |          |            |       |    |               |        |

TABLE XXVII—Continued.

| Date | C  | $\delta_1$ | Cylinder | $\delta_2$ | N  | W | v | Mean   |
|------|----|------------|----------|------------|----|---|---|--------|
| 1906 |    |            |          |            |    |   |   |        |
| Nov. | 14 | 18.168     | 1        | 152.849    | 4  | 9 | 4 | 2.9961 |
|      | 15 | 18.181     | 2        | 152.864    | 4  | 8 | 4 | 2.9959 |
|      | 15 | 18.186     | 2        | 152.866    | 3  | 8 | 4 | 2.9959 |
|      | 15 | 18.182     | 1        | 152.873    | 3  | 8 | 4 | 2.9959 |
|      | 15 | 19.300     | 2        | 152.922    | 3  | 6 | 3 | 2.9954 |
|      | 15 | 18.199     | 3        | 152.884    | 3  | 6 | 3 | 2.9957 |
|      | 15 | 18.188     | 1        | 152.871    | 3  | 4 | 2 | 2.9959 |
|      | 16 | 18.216     | 1        | 152.869    | 5  | 6 | 3 | 2.9959 |
|      | 16 | 18.215     | 1        | 152.873    | 5  | 6 | 3 | 2.9959 |
|      | 17 | 18.208     | 2        | 152.847    | 5  | 6 | 3 | 2.9961 |
|      | 17 | 18.205     | 1        | 152.857    | 4  | 7 | 3 | 2.9960 |
|      | 17 | 18.205     | 2        | 152.869    | 5  | 8 | 4 | 2.9959 |
|      | 17 | 18.207     | 1        | 152.855    | 6  | 6 | 3 | 2.9960 |
|      | 17 | 18.197     | 2        | 152.840    | 9  | 8 | 3 | 2.9962 |
|      | 17 | 18.195     | 4        | 152.854    | 5  | 4 | 2 | 2.9960 |
| Dec. | 4  | 16.799     | 2        | 152.793    | 2  | 6 | 3 | 2.9966 |
|      | 4  | 16.797     | 2        | 152.792    | 5  | 6 | 3 | 2.9966 |
|      | 4  | 16.806     | 8        | 152.806    | 3  | 4 | 2 | 2.9965 |
|      | 4  | 16.855     | 4        | 152.806    | 10 | 8 | 3 | 2.9965 |
|      | 5  | 18.117     | 2        | 152.814    | 2  | 4 | 2 | 2.9964 |
|      | 6  | 18.563     | 3        | 152.792    | 2  | 6 | 3 | 2.9966 |
|      | 6  | 18.565     | 2        | 152.792    | 6  | 6 | 3 | 2.9966 |
|      | 6  | 18.562     | 1        | 152.797    | 4  | 8 | 4 | 2.9966 |
|      | 6  | 18.559     | 2        | 152.818    | 8  | 6 | 3 | 2.9964 |
|      | 7  | 18.534     | 10       | 152.833    | 19 | 6 | 1 | 2.9962 |
|      | 7  | 18.528     | 10       | 152.853    | 9  | 6 | 2 | 2.9961 |
|      | 7  | 18.494     | 1        | 152.836    | 2  | 4 | 2 | 2.9962 |
| 1907 |    |            |          |            |    |   |   |        |
| Jan. | 5  | 17.799     | 1        | 152.790    | 4  | 8 | 4 | 2.9967 |
|      | 5  | 17.768     | 1        | 152.790    | 4  | 6 | 3 | 2.9967 |
|      | 5  | 17.770     | 2        | 152.788    | 4  | 6 | 3 | 2.9967 |
| Feb. | 4  | 20.619     | 0        | 152.878    | 6  | 7 | 3 | 2.9958 |
|      | 4  | 20.619     | 0        | 152.890    | 3  | 6 | 3 | 2.9957 |
|      | 4  | 20.617     | 2        | 152.884    | 7  | 8 | 4 | 2.9957 |
|      | 4  | 20.612     | 1        | * 76.745   | 2  | 6 | 3 | 2.9957 |
|      | 4  | 20.604     | 1        | * 76.752   | 2  | 6 | 3 | 2.9957 |
|      | 4  | 20.608     | 1        | * 76.758   | 2  | 6 | 3 | 2.9957 |

TABLE XXVII—Continued.

| Date   | C      | $\delta_1$ | Cylinder | $\delta_2$ | N | W | v      | Mean    |
|--------|--------|------------|----------|------------|---|---|--------|---------|
| 1907   |        |            |          |            |   |   |        |         |
| Feb. 4 | 20.615 | 1          | 152.879  | 3          | 6 | 3 | 2.9958 | 2.99596 |
| 5      | 20.627 | 1          | 152.835  | 2          | 6 | 3 | 2.9962 |         |
| 5      | 20.627 | 2          | 152.854  | 4          | 6 | 3 | 2.9960 |         |
| 5      | 20.629 | 1          | 152.876  | 4          | 6 | 3 | 2.9958 |         |
| 5      | 20.632 | 1          | 152.885  | 8          | 6 | 2 | 2.9957 |         |
| 5      | 20.639 | 2          | 152.843  | 2          | 6 | 3 | 2.9962 |         |
| 5      | 20.631 | 2          | 152.851  | 4          | 6 | 3 | 2.9961 |         |
| 7      | 20.409 | 3          | 152.854  | 4          | 8 | 4 | 2.9960 |         |
| 7      | 20.409 | 1          | 152.864  | 3          | 6 | 3 | 2.9959 |         |
| 7      | 20.411 | 1          | 152.867  | 3          | 6 | 3 | 2.9959 |         |
| 7      | 20.419 | 1          | 152.864  | 3          | 6 | 3 | 2.9959 |         |
| 7      | 20.421 | 1          | 152.864  | 3          | 7 | 3 | 2.9959 |         |

\* The middle section of the condenser consisted of section three only.

The average deviation of the results in the eighth column of the table from this mean is

$$0.00030 \times 10^{10}, \text{ or } 10 \text{ parts in } 100,000$$

The average value of  $\delta_2$  for the remainder of the observations in this table is 0.0046 micro-microfarads. The weighted mean of these 329 independent determinations is

$$v = 2.99609 \times 10^{10}$$

and the average deviation from this mean is

$$0.00027 \times 10^{10}, \text{ or } 9 \text{ parts in } 100,000$$

The mean of all the values as they stand is  $v = 2.99614 \times 10^{10}$ ; this, however, gives undue weight to the results obtained on those days on which several series of observations were taken. These series are all independent so far as the electromagnetic measurements go, but are mere duplicates so far as the electrostatic capacity is concerned. Hence it seems more fair to give to the mean result of any one day the mean weight attached to the separate series; thus from the daily means we find

$$v = 2.99624 \times 10^{10}$$

The average variation of the daily means from this is

$$0.00027 \times 10^{10}, \text{ or } 9 \text{ parts in } 100,000$$

## 56. RESULTS BY THE PLATE CONDENSERS.

As already stated the importance of the correct adjustment of the plates to parallelism, as well as the importance of the lower plate being truly plane, was not recognized until the conclusion of the work, hence all results obtained are in error from these causes. Also, until near the end of the work (October, 1906), the importance of the plate being strictly in the plane of the guard ring was not recognized. As a consequence the adjustment was but approximate and the error due to this cause was large. Later the adjustment was made with more care, but it was not possible to make the error from this cause negligible.

Consequently, all results obtained by use of the plates have to be corrected for these two sources of error.

By dividing the results into a series of groups such that each group covers a period over which the adjustment of the plates was not knowingly altered, and by assuming that the adjustment (so far as the displacement of the plate from the plane of the guard ring, and the relative inclination of the plates are concerned) remained unchanged throughout each group, we can determine the errors in adjustment which must be assumed in order to bring into accord with an assumed value of  $v$  and with one another the results obtained at different distances between the plates.

Proceeding in this manner we have found that all the results obtained can be made concordant with one another and with the value of  $v$  given by the other condensers by the assumption of very probable errors in these two adjustments. Until April 17, 1906, there was no attempt made to maintain the temperature of the condenser constant. As a consequence, the results before this date are more variable than the others, and it seems scarcely worth while to present them in detail since the essential features of the uncorrected results and of the corrections are sufficiently shown by the later and more reliable work. We may, however, state that the 66 determinations made before this date can be made to give a mean of 2.99627 by assuming displacements of the plate not exceeding  $68\ \mu$ , and inclinations not exceeding  $4' 42''$ . The capacity was varied in the ratio of 1 to 15, and the average deviation from the mean given above is but 6 in 100,000.

The other results are set forth in Tables XXVIII and XXIX.

TABLE XXVIII.

Results by Plate Condenser. Plate A.

(Temperature controlled.)

| Date            | Distance between the plates in mm |          |          |          |          |         |         |         |
|-----------------|-----------------------------------|----------|----------|----------|----------|---------|---------|---------|
|                 | 1.64                              | 2.64     | 3.64     | 4.64     | 5.64     | 6.64    | 7.64    | 9.64    |
| 1906            |                                   |          |          |          |          |         |         |         |
| Apr. 17.....    | 2.9971                            | 2.9972   |          |          |          |         |         |         |
| 25.....         |                                   | 2.9966   |          |          |          |         |         |         |
| 26.....         |                                   | 2.9969   |          |          |          |         |         |         |
| 28.....         |                                   | 2.9967   |          |          |          |         |         |         |
| 28.....         | 2.9976                            | 2.9965   |          |          |          |         |         |         |
| 30.....         |                                   | 2.9971   |          |          |          |         |         |         |
| May 1.....      |                                   | 2.9969   |          |          |          |         |         |         |
| 1.....          |                                   | 2.9963   |          |          |          |         |         |         |
| 5.....          | 2.9972                            | 2.9967   | 2.9966   |          |          |         |         |         |
| 9.....          |                                   |          | 2.9954   | 2.9951   | * 2.9939 |         |         |         |
| 9.....          |                                   |          | 2.9964   |          | 2.9946   | 2.9952  | 2.9943  | 2.9928  |
| 9.....          |                                   |          |          |          | 2.9957   |         | 2.9948  |         |
| 11.....         |                                   | 2.9967   | 2.9967   |          | 2.9957   |         | 2.9947  | 2.9937  |
| 11.....         |                                   |          | 2.9960   |          | 2.9951   |         | 2.9946  | 2.9946  |
| Mean .....      | 2.99730                           | 2.99676  | 2.99622  | 2.9951   | 2.99528  | 2.9952  | 2.9946  | 2.9937  |
| Correction..... | -0.00143                          | -0.00041 | +0.00028 | +0.00077 | +0.00115 | +0.0014 | +0.0017 | +0.0020 |
| v.....          | 2.99587                           | 2.99635  | 2.99650  | 2.99587  | 2.99643  | 2.9966  | 2.9963  | 2.9957  |

Mean of all v=2.99626 × 10<sup>10</sup>  
p<sub>1</sub>=0.64 mm; q-p=66.5 μ; δ=94.5 μ; θ=3' 15''

\* Omitted from mean.

In Table XXVIII are contained the results obtained by the use of Plate A, when the temperature of the condenser was maintained constant. These fall into a single group and but one initial distance,  $p_1=0.64$  mm, was employed. Hence the uncorrected values have been tabulated and averaged and the corrections are applied to the means. Though the uncorrected means range from 2.9973 to 2.9937, it is necessary to assume a displacement of but 66.5 μ and an inclination of but 3' 15'' in order to make them range but from 2.9966 to 2.9957, and now the high and low values alternate while the uncorrected values steadily decreased as the capacity was reduced. The mean of all these results is  $2.99626 \times 10^{10}$ .

TABLE XXIX.  
Results by Plate Condenser.—Plate B.

| Date                                                                    | p <sub>i</sub><br>mm | p-q<br>μ | δ<br>μ | θ      | 1       |      |    | 2       |     |      | 3       |     |       | 4       |     |    | 5       |     |    | 10      |    |    | 15      |    |    | 20      |    |      | 25 |         |    |
|-------------------------------------------------------------------------|----------------------|----------|--------|--------|---------|------|----|---------|-----|------|---------|-----|-------|---------|-----|----|---------|-----|----|---------|----|----|---------|----|----|---------|----|------|----|---------|----|
|                                                                         |                      |          |        |        | O       | C    | V  | O       | C   | V    | O       | C   | V     | O       | C   | V  | O       | C   | V  | O       | C  | V  | O       | C  | V  | O       | C  | V    | O  | C       | V  |
| 1906.                                                                   |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| May 11                                                                  | 0.68                 | 170      | 161    | 5' 32" |         |      |    | 58      | +2  | 60   | 45      | +18 | 63    |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 25                                                                      |                      |          |        |        |         |      |    | 56      | +2  | 58   | 46      | +18 | 64    |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 25                                                                      |                      |          |        |        |         |      |    | 69      | +2  | 71   |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 29                                                                      |                      |          |        |        |         |      |    | 47      | +2  | * 49 | 78      | +18 | * 96  |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 29                                                                      |                      |          |        |        |         |      |    | 65      | +2  | * 67 | 72      | +18 | * 100 |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 29                                                                      |                      |          |        |        |         |      |    |         |     |      | 58      | +18 | * 76  |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| Oct. 3                                                                  | 0.66                 | 2.7      | 58     | 2' 0"  | 71      | — 8  | 63 |         |     |      |         |     |       | 69      | — 3 | 66 |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 8                                                                       |                      |          |        |        | 72      | — 8  | 64 |         |     |      |         |     |       | 64      | — 3 | 61 |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 1907.                                                                   |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| Jan. 5                                                                  | 0.59                 | 5.3      | 37.5   | 1' 17" | 67      | — 4  | 63 | 65      | — 2 | 63   | 56      | — 1 | 55    |         |     |    | 63      | +0  | 63 | 69      | +2 | 71 | 53      | +3 | 56 | 31      | +4 | * 35 | 45 | +4      | 49 |
| 8                                                                       |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    | 61      | +0  | 61 | 63      | +2 | 65 | 75      | +3 | 78 | 72      | +4 | 76   | 70 | +4      | 74 |
| 10                                                                      |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 10                                                                      |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 18                                                                      |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 18                                                                      |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 24                                                                      | 0.58                 | 10.6     | 73.0   | 2' 31" | 85      | — 17 | 68 |         |     |      |         |     |       |         |     |    | 66      | — 2 | 64 | 63      | +2 | 65 | 60      | +4 | 64 | 48      | +5 | 53   | 41 | +6      | 47 |
| 24                                                                      |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 25                                                                      |                      |          |        |        | 74      | — 17 | 57 |         |     |      |         |     |       |         |     |    | 65      | — 2 | 63 | 66      | +2 | 68 |         |    |    |         |    |      |    |         |    |
| 25                                                                      |                      |          |        |        | 68      | — 5  | 63 |         |     |      |         |     |       |         |     |    | 59      | +2  | 61 | 50      | +5 | 55 |         |    |    |         |    |      |    |         |    |
| 28                                                                      | 0.60                 | 13.3     | 47.4   | 1' 38" | 78      | — 5  | 73 |         |     |      |         |     |       |         |     |    | 62      | +2  | 64 | 56      | +5 | 61 | 41      | +7 | 48 | 40      | +8 | 48   |    |         |    |
| Feb. 1                                                                  |                      |          |        |        | 57      | — 5  | 52 |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 1                                                                       |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| 1                                                                       |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |
| Mean corrected value                                                    |                      |          |        |        | 2.99628 |      |    | 2.99630 |     |      | 2.99607 |     |       | 2.99635 |     |    | 2.99626 |     |    | 2.99641 |    |    | 2.99615 |    |    | 2.99570 |    |      |    | 2.99558 |    |
| Mean omitting three smallest capacities is v=2.99630×10 <sup>10</sup> . |                      |          |        |        |         |      |    |         |     |      |         |     |       |         |     |    |         |     |    |         |    |    |         |    |    |         |    |      |    |         |    |

\* Omitted from mean.

The results obtained by the use of Plate B are contained in Table XXIX. The temperature throughout each day's work was maintained constant. In the first column is given the date, then follows  $p_1$ , in mm, the initial distance between the plates as calculated from the measured electromagnetic capacity and the assumed value  $v = 2.9963 \times 10^{10}$ . In the third column is given, in microns, the distance  $q-p$  by which the plane of the plate is below the plane of the ring; in the next column is given, in microns, the maximum distance  $\delta$  of the rim of the collector plate from the imaginary plane through its center and parallel to the movable plate; at least, this would be the value of  $\delta$  were the lower plate plane; as it is somewhat cylindrical in form the value of  $\delta$  is greater than that given by reference to the equivalent mean plane of the lower plate. In the following column is given the effective inclination of the plates,  $\theta = \tan^{-1} \frac{\delta}{R}$

The numbers at the heads of the sixth and succeeding columns denote the approximate distance, in mm, between the plates. Each of these columns is divided into three parts; in the first (O) is contained the fourth and fifth significant figures of the values of  $v$  as deduced from the observations without applying the correction for the inclination and the displacement of the collector plate; the middle portion contains the correction (C) due to these causes, and the third portion contains the fourth and fifth significant figures of the corrected values of  $v$ . The first three significant figures of  $v$  are in every case 299. At the bottom of the table are given the averages of the corrected values corresponding to each distance between the plates, values marked with an asterisk being omitted. These means agree very well with one another, though the capacities measured varied in the ratio of 1 to 10. Their mean is  $v = 2.99630 \times 10^{10}$ .

From this table it is easy to see the advantage of the pivoted lever (page 454) in the adjustment of the plates. In May the plate was adjusted by the use of the eye and fingers only, and the displacement of the plate was 170 microns, and the inclination was  $5' 32''$ . On October 3, January 5, 24, and 28, the plate was displaced and readjusted by the use of the lever. Under these conditions the displacement varied but from 3 to 13 microns, and the inclination from  $1' 17''$  to  $2' 31''$ .

That the values obtained from the smaller capacities are somewhat erratic is not surprising. If we assume that the uncertainty in the electromagnetic measurements is 0.01 micro-microfarad (this is but two-thirds of the average deviation of the measured capacity of the cylindrical condenser from its mean on the same day, page 591), then the uncertainty introduced by this cause alone is as follows:

|                         |     |     |     |     |      |      |                      |
|-------------------------|-----|-----|-----|-----|------|------|----------------------|
| Distance between plates | 1   | 5   | 10  | 15  | 20   | 25   | 30 mm.               |
| Uncertainty in $\nu$    | 0.5 | 2.7 | 5.4 | 8.4 | 10.8 | 13.5 | $15.3 \times 10^6$ . |

That is, at 20 mm distance between the plates the values found for  $\nu$  may vary between 2.9974 and 2.9952, even when the uncertainty in the electromagnetic measurements amounts to but 0.01 micro-microfarad, or a capacity of 90 microns.

Though the results obtained from the plates are of no value in establishing the value of  $\nu$ , they are not at all discordant with the results obtained by the use of the other condensers. Realizing now the importance of these adjustments we shall take pains when we return to this work to reduce these errors to a minimum, and trust then to be able to obtain, by the use of plates, results of positive value.

## 57. DISCUSSION OF RESULTS.

From what has been already stated it is evident that the results obtained by the use of the plates and of the cylinders without guards are of a negative value—they do not contradict the conclusions that may be drawn from the results obtained by the use of the other condensers.

The results obtained by the use of the spheres and of the guard cylinders and contained in Tables XXI, XXII, and XXVII are shown graphically in Fig. 44. Each value of  $\nu$  given in these tables is represented by a dot, of which the size denotes the weight attached to this value, and the ordinate gives the value of  $\nu$  corresponding to it. The dots are in order of the corresponding determinations, from left to right. The dotted lines represent the values of  $\nu$  corresponding to the weighted means of the various groups.

These results are also summarized in Table XXX, where  $\Delta$  is the average variation of the members of each group from their mean,

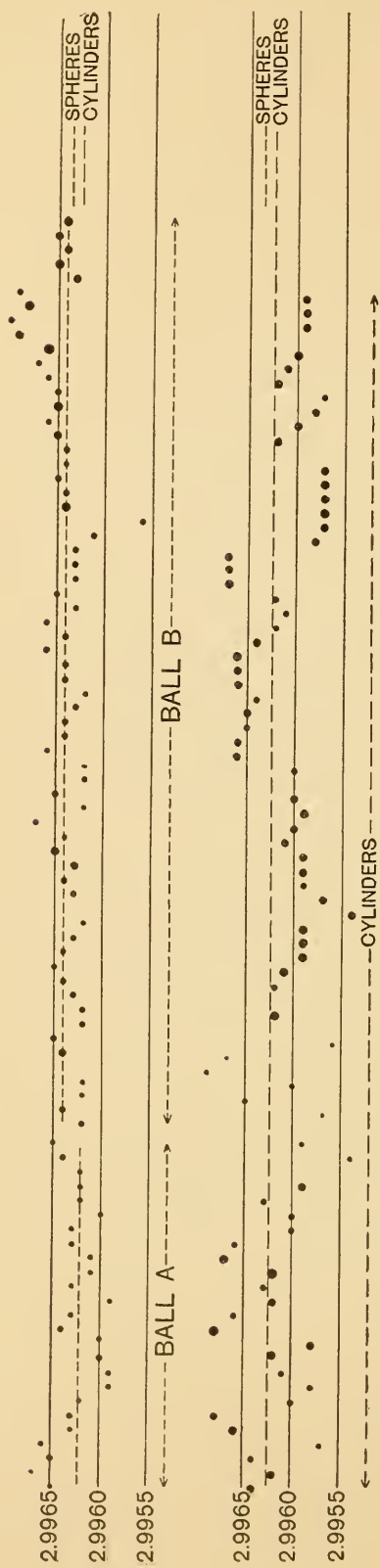


Fig. 44.—Graphical Summary of Results.

$\Delta_1$  is the corresponding average variation from the weighted mean of all (2.99629), N is the number of independent determinations.

TABLE XXX.

| Date.         | Condenser. | V                        | $\Delta$                 | N   | $\Delta_1$               |
|---------------|------------|--------------------------|--------------------------|-----|--------------------------|
| 1905          | A          | $2.99621 \times 10^{10}$ | $0.00018 \times 10^{10}$ | 64  | $0.00018 \times 10^{10}$ |
| 1905          | B          | 2.99637 "                | 0.00011 "                | 146 | 0.00013 "                |
| 1906          | B          | 2.99659 "                | 0.00017 "                | 89  | 0.00030 "                |
| 1906          | Cylinders  | 2.99624 "                | 0.00027 "                | 603 | 0.00028 "                |
| Weighted Mean |            | 2.99629 "                | Mean                     |     | 0.00022 "                |

The abnormally high value obtained by the use of ball B in 1906 has already been noticed. Being based upon the observations of but five days, it is not to be given undue importance, but nevertheless it does indicate a possible outstanding uncertainty of some kind.

From the concordance of the other observations we believe that the mean value (obtained from a series of observations extending over several months) of the square root of the ratio of the electrostatic to the electromagnetic capacity of an air condenser is

$$v = 2.9963 \times 10^{10} \left[ \frac{\text{cm Int} \Omega}{\text{sec}} \right]^{\frac{1}{2}}$$

with an uncertainty of not more than 1 in 10,000.

Since the mean temperature at which the work was done was about 20°0, this gives us for the value reduced to vacuo (assuming the dielectric constant of air at 20°0 C and 760 mm pressure to be 1.00055)

$$v_0 = 2.9971 \times 10^{10}$$

We have given above a very full discussion of the sources of error in the determination of capacity in electrostatic and in electromagnetic measure, partly for the sake of the present work and partly because we think it desirable that the accuracy of the work be carried further in the near future. As we have already stated, the accuracy we expected to attain when the work was taken up nearly three years ago was considerably short of what we believe we have actually accomplished. We then regarded errors negligible

which we later took into account, and much of the theoretical investigation of the sources of error has been made since the observations were completed. Hence we were satisfied to begin the work with condensers much less perfect mechanically than they can be made. We are unable to pursue the subject any further at the present time, on account of other urgent work. Indeed, owing to the uncertainty in the absolute value of the ohm, there is no present need of greater accuracy than we believe we have attained. When, however, the ohm becomes known, to, say, 1 part in 15,000, as it doubtless will at an early date, we believe it would be a profitable undertaking to carry out a new determination of  $v$ , using spherical, cylindrical, and plate condensers, constructed with all possible accuracy, and to try to obtain a result trustworthy to within one unit in the fifth place—that is, to 1 part in 30,000. That would of course require improvements in the determination of electromagnetic capacity as well as improvements in the design and construction of the condensers. Such improvements are practicable, and such an accuracy, we think, is attainable. We hope we may be able to return to this problem in the near future.

#### 58. PREVIOUS RESULTS AND THE VELOCITY OF LIGHT.

A résumé of all former work upon this subject was prepared by H. Abraham<sup>21</sup> and presented before the International Congress of Physics which met in Paris in 1900. At the conclusion of this report he states that he considers the most accurate results to be the following:

|                         |                         |
|-------------------------|-------------------------|
| Himstedt.....           | $3.0057 \times 10^{10}$ |
| Rosa.....               | $3.0000 \times 10^{10}$ |
| Thomson and Searle..... | $2.9960 \times 10^{10}$ |
| H. Abraham.....         | $2.9913 \times 10^{10}$ |
| Pellat.....             | $3.0092 \times 10^{10}$ |
| Hurmuzescu.....         | $3.0010 \times 10^{10}$ |
| Perot and Fabry.....    | $2.9978 \times 10^{10}$ |
| Mean.....               | $3.0001 \times 10^{10}$ |

Abraham stated that it was *probable* that this mean differs from the true value by not more than 1 in 1,000, but that this is not cer-

<sup>21</sup> Congr s International de Physique, 1900; Rapports II, pp. 247-267.

tain. All of the above results are referred to air. Of these the result obtained by Thomson and Searle is very near the value given by the present work, which is a little more than 1 in 1,000 less than the above mean.

Turning to the question of the velocity of light, we have the results of Michelson<sup>23</sup>, obtained by the revolving-mirror method and giving

$$v = 299853 \text{ km} \pm 60 \text{ km},$$

that of Newcomb<sup>24</sup>, obtained by the same method and giving

$$v = 299860 \text{ km} \pm 30 \text{ km},$$

and that by Perrotin<sup>25</sup>, obtained by the toothed-wheel method and giving

$$v = 299860 \text{ km} \pm 80 \text{ km}$$

also, Weinberg<sup>26</sup> in discussing the results reaches the conclusion that the most probable value of the velocity of light in vacuo is

$$v = 299852 \text{ km} \pm 24 \text{ km}$$

From which we may conclude that the velocity of light in vacuo is

$$v = 2.9986 \times 10^{10} \frac{\text{cm}}{\text{sec}}$$

with an accuracy of 1 in 10,000.

This differs from our value for  $v_0$  by 5 parts in 10,000, with a possible uncertainty of 2 parts in 10,000. Whether the remainder of the difference (1 in 3,000) is due to uncertainty in the value of

<sup>23</sup> Astron. papers of the Am. Ephemeris, 1, pp. 112-145, 1880; 2, pp. 231-258, 1885.

<sup>24</sup> Astron. papers of the Am. Ephemeris, 2, pp. 107-230, 1883.

<sup>25</sup> Comptes Rendus, 135, pp. 881-884, 1902.

<sup>26</sup> Beibl. 28, p. 613, 1904.

the international ohm, or to greater errors in our determination of  $v$  and in the measurement of the velocity of light than we have supposed, or whether there is really a small difference between the velocity of light and the ratio of the units as measured by the method and at the frequencies we have employed, remains for future work to explain.

WASHINGTON, May 20, 1907.











